Do five of the following six problems. Write each answer on a separate piece of paper.

1. Define the following terms:
   (a) finite state automaton
   (b) regular language
   (c) Given a finite set Σ, define Σ*
   (d) Given a string s, define |s|
   (e) Given string s₁, s₂, define s₁ ◦ s₂

2. Find the error in the following proof that 2 = 1.
   Consider the equation a = b. Multiply both sides by a to obtain a² = ab. Subtract b² from both sides to get a² − b² = ab − b². Now factor each side, (a + b)(a − b) = b(a − b), and divide each side by (a − b), to get a + b = b. Finally, let a and b equal 1, which shows 2 = 1.

3. Give the state diagrams of NFAs recognizing the following languages. In all cases the alphabet is Σ = {a, b, c, d, . . . , x, y, z}, the 26 lowercase letters.
   (a) \{w | w contains the substring yellow\}
   (b) \{w | w is of even length or ends with the substring bye\}

4. Prove that the class of regular languages is closed under the star operator.

5. Let Σ = {0, 1, +, =}. Prove that the following language is not regular:
   \[ADD = \{x = y + z | x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}\]

6. Give context-free grammars generating the following languages. The language is Σ = \{a, b\}.
   (a) \{w | w starts and ends with the same symbol\}
   (b) the complement of the language \{aⁿbⁿ | n ≥ 0\}