Do five of the following six problems. Write each answer on a separate piece of paper.

1. Define the following terms:
   (a) regular expression
   (b) Pigeonhole Principle
   (c) stack
   (d) Given a string $s$, define $|s|
   (e) Given finite sets $\Sigma_1, \Sigma_2$, define $\Sigma_1 \circ \Sigma_2$

2. Find the error in the following proof that all horses are the same color.
   CLAIM: In any set of $h$ horses, all horses are the same color.
   PROOF: By induction on $h$.
   Basis: For $h = 1$. In any set containing just one horse, all horses clearly are the same color.
   Induction step: For $k \geq 1$, assume that the claim is true for $h = k$ and prove that it is true for $h = k + 1$. Take any set $H$ of $k + 1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set $H_1$ with just $k$ horses. By the induction hypothesis, all the horses in $H_1$ are the same color. Now replace the removed horse and remove a different one to obtain the set $H_2$. By the same argument, all horses in $H_2$ are the same color. Therefore, all the horses in $H$ must be the same color, and the proof is complete.

3. Give the state diagrams of NFAs recognizing the following languages. In all cases the alphabet is $\Sigma = \{a, b, c, d, \ldots, x, y, z\}$, the 26 lowercase letters.
   (a) $\{w \mid w$ contains the substring $help\}$
   (b) $\{w \mid w$ is of length at least 2 and an even numbers of $z$’s\}$

4. Prove that the class of regular languages is closed under the union operator.

5. Prove that the following language is not regular:
   $\{w \mid w \in \{0, 1\}^* \text{ is not a palindrome}\}$

6. Give context-free grammars generating the following languages:
   (a) $\{w \mid \text{the length of } w \text{ is odd and its middle symbol is 0}\}$
   (b) The complement of the language $\{a^n b^n \mid n \geq 0\}$