Do five of the following six problems. Write each answer on a separate piece of paper.

1. Define the following terms:
   (a) regular language
   (b) stack
   (c) Given a finite set $\Sigma$, define $\Sigma^*$
   (d) Given a string $s$, define $|s|$
   (e) Given finite sets $\Sigma_1, \Sigma_2$, define $\Sigma_1 \circ \Sigma_2$

2. Find the error in the following proof that $2 = 1$.
   Consider the equation $a = b$. Multiply both sides by $a$ to obtain $a^2 = ab$. Subtract $b^2$ from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side, $(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$, to get $a + b = b$. Finally, let $a$ and $b$ equal 1, which shows $2 = 1$.

3. Give the state diagrams of NFAs recognizing the following languages. In all cases the alphabet is $\Sigma = \{a, b, c, d, \ldots, x, y, z\}$, the 26 lowercase letters.
   (a) $\{w \mid w \text{ contains the substring } \text{blue}\}$
   (b) $\{w \mid w \text{ is of even length and begins with the substring } \text{hi}\}$

4. Prove that the class of regular languages is closed under the union operator.

5. Prove that the following language is not regular:
   The complement of $\{0^n1^n \mid n \geq 0\}$

6. Give context-free grammars generating the following languages. Both are over the alphabet $\Sigma = \{a, b\}$.
   (a) $\{w \mid w = w^R, \text{ that is } w \text{ is a palindrome}\}$
   (b) The set of strings with twice as many a’s as b’s.