VICTOR PAN'S RESEARCH IN 1962 – 2017 ON TWO PAGES

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Below I outline Victor Pan's accomplishments up to 2017, beginning with his seminal work of 1962 where he proved *optimality of Horner's polynomial evaluation*, by introducing novel techniques, now classical. His large survey of 1966 covering also his faster algorithms for polynomial evaluation with pre-processing attracted to the area S. Winograd and V. Strassen and led to emergence of a new field of *Algebraic Computational Complexity*, now popular.

His research breakthrough in 1978 on *fast matrix multiplication* became known worldwide and inspired fast progress in the field after almost a decade of stalemate. His novel trilinear aggregation was an ingredient of all further progress on this subject since 1979 and was pioneering in application of tensor decomposition to matrix computations, now a thriving computational area. In 1982 he reached exponent 2.7734 of the complexity of feasible n-by-n matrix multiplication (for $n < 1,000,000$) - it is still smallest known. The implementation of his algorithms by I. Kaporin in 2004 is superior to Strassen's and the other known fast algorithms in both memory usage and numerical stability.

The following quotation from Donald E. Knuth is with his permission:

"I am convinced that his research on matrix multiplication was the most outstanding event in all of theoretical computer science during 1978. The problem he solved, to multiply $n \times n$ matrices with less than $O(n^{\lg 7})$ operations, was not only a famous unsolved problem for many years, it also was worked on by all of the leading researchers in the field, worldwide. Pan's breakthrough was based on combination of brilliant ideas, and there is no telling what new avenues this will open."

Pan's *hierarchical aggregation* (devised jointly with Miranker, 1980) was a basic step of the emergence of *Algebraic Multigrid*, now a popular field.

His and his coauthors' novel *fast and processor efficient parallel algorithms* of 1985-2000 for computations with matrices, polynomials, and graphs have greatly advanced the field.

His surprising *transformations of matrix structures* of 1989 unified computations with matrices having displacement structure. Dramatic decrease of the complexity of Cauchy-like matrix computations was implied immediately and further extensions later: by dozens of experts since 1995 to the design of the user's choice algorithms for Toeplitz linear systems (omnipresent in modern computations) and by himself in 2013-2017 to dramatic acceleration of some fundamental numerical computations with structured matrices and polynomials. New links of 2016 by Beckermann and Townsend to matrices with rank structure promise new extensive practical algorithmic applications.

In 1995, by extending the divide-and-conquer algorithms of Schönhage, and Neff and Reif, he solved the classical, four millennia old and still highly important problem of *polynomial root-finding* in record low arithmetic and Boolean time, optimal up to poly-logarithmic factors. His algorithms approximate all roots almost as fast as one can read the input coefficients. In view of recent work, his 2000 *acceleration of Weyl's quad-tree algorithm* promises extension to more practical nearly optimal root-finders. His many other results on root-finding (some in 2015-17) were also highly recognized, e.g., on real root-finding and real and complex root-refining. In 2013 he co-authored J.M. McNamee in publishing a book with *unique nearly complete coverage of the known root-finders up to the date.*

In 1997-2005, with B. Mourrain and with I.Z. Emiris he proposed a number of novel and presently popular structured matrix methods for solving a *multivariate polynomial system of equations.* With

Mourrain he asymptotically accelerated the known multivariate polynomial root-finders in STOC 1998 and SICOMP 2003 and received the Best Paper Award of Journal of Complexity 2000.

Motivated by advancing computer technology he proposed new insight and novel techniques for replacing pivoting with randomization in *Gaussian elimination*, with extension to *low-rank approximation* of a matrix, which is a highly popular subject. Moreover, in 2016 he reported dramatic decrease of the known record bound on the arithmetic complexity of the latter highly important and popular computational problem (from quadratic to nearly linear time bound for the average case input). He confirmed efficiency of his analysis with test results for real world data.

In 2013 he was designated for a *Fellowship in the American Mathematical Society* for "Contributions to the Mathematical Theory of Computation" with his novel ideas, insights, techniques and algorithms. He keeps prolific and influential through 2017, supporting strongly progress in *Symbolic and Numerical Computations* and the design of *Randomized Matrix Algorithms.*

SUMMARY OF CONTRIBUTIONS

Within more than 5 decades of his prolific work, Victor Pan made groundbreaking contributions at the foundation and technical levels of Information Science and technology. He introduced a number of new insights and novel methods in Computer Science and Computational Mathematics, greatly enhanced efficiency of the known algorithms, revealed unexpected links among some seemingly distant subjects, and proposed new research directions and new areas of study.

Google Scholar and DBLP show his 4 books (1623+LXXIV pages overall), over 20 surveys in journals and book chapters, over 170 research articles in leading journals, and over 80 in refereed conference proceedings such as STOC, FOCS, SODA, ICALP, SPAA, CSR, ISSAC, and SNC.

Some concepts and definitions introduced in his papers (e.g., active operations and basic substitution) as well as his techniques and insights are commonly used, sometimes as folklore. His research, publications and service in the field helped establish synergistic links among various areas of computing, e.g., theoretical computer science, symbolic computations, numerical computations, and applied linear algebra.

Victor Pan has served his profession in various ways, in particular by his groundbreaking research and publications. He was the founder of the popular field of *Algebraic Complexity of Computations* and one of the founders of *Symbolic-Numerical Computations* and *Algebraic Multigrid*, helped establishing other new areas of study and linking various areas of study to the benefits of each other.

PROFESSIONAL RECOGNITION OF ACHIEVEMENTS IN COMPUTER SCIENCE AND COMPUTATIONAL MATHEMATICS

- 2000: Best Paper Award (with Bernard Mourrain), Journal of Complexity
- 2013: Designation of Fellowship in American Math Society for "Contributions to the Mathematical Theory of Computation"
- NSF Grants 1980-2020, including Special Creativity Extension Award in 1993 from Numeric, Symbolic, and Geometric Computation Program of the CCR Division in the Directorate CISE.
- Extensive citation in journals and books, and enthusiastic reviews and citations by experts in journals and magazine articles. His technique of active operations/linear substitution was cited in two well-known articles of Volker Strassen as *Pan's method*. D. E. Knuth's Volume 2 (Seminumerical Algorithms) of The Art of Computer Programming cites Pan's works (together with R. Brent's) most extensively among all cited authors. D.E. Knuth specifically praised Pan's work on matrix multiplication as *"the most outstanding event in all of theoretical computer science during 1978."* In June 1995 SIGACT NEWS used the same adjective *"outstanding"* for his book of 1994 with Bini. Some of Pan's work has been covered in the magazines Byte, Science, Science News, and American Scientist.

CITATION: Victor Pan was the founder of the field of Algebraic Computational Complexity and co-founder of Symbolic-Numerical Computations and Algebraic Multigrid, now highly popular, and for over 50 years has been continuously recording high-impact, seminal contributions to the two former subject areas.