Do five of the following six problems. Write each answer on a separate piece of paper.

1. (a) Define *decidable set* and give an example of a set that is not decidable.
   (b) Assume that the alphabet and tape alphabet are: $\Sigma = \Gamma = \{0, 1\}$. Give an implementation level description of a Turing machine that decides the language:
   \[ \{w \mid w \text{ is a palindrome}\} \]

2. (a) Show that the set of all positive rational numbers is countable.
   (b) Show that the set of all positive real numbers is uncountable.

3. (a) Show that the Halting Problem is undecidable, using the diagonalization method.
   (b) Show that the set of all totally defined computable functions is not decidable.

4. (a) Show that the set of Turing-recognizable languages is closed under concatenation.
   (b) Show that if $A$ is decidable, then $A$ and $\overline{A}$ are Turing-recognizable.

5. (a) State the Post Correspondence Problem (PCP).
   (b) Show that PCP is decidable over the alphabet $\Sigma = \{1\}$.

6. (a) Show that $\leq_m$ is transitive.
   (b) Show for all $A$, $B$, there exists a set $J$ such that $A \leq_T J$ and $B \leq_T J$. 