Symmetry Project

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BACKGROUND: Euclidean Axioms, Midpoints, Perpendicular Bisectors, Congruent Triangles.

LINE SYMMETRY: *Plot points* $A = (2, 3)$ $A' = (-2, 3)$ $B = (5, 0)$, $B' = (-5, 0)$, $C = (4, -2)$ *and* $C' = (-4, -2)$ *and observe that triangles* ΔABC *and* $\Delta A'B'C'$ *are "symmetric" with respect to the y axis. Note that the y-axis is the perpendicular bisector of* $\overline{AA'}$ *and of* $\overline{BB'}$ *and of* $\overline{CC'}$.

LINE OF SYMMETRY THEOREM: Let L be a line and let ΔABC and $\Delta A'B'C'$ be triangles such that L is the perpendicular bisector of $\overline{AA'}$, L is the perpendicular bisector of $\overline{BB'}$, and L is the perpendicular bisector of $\overline{CC'}$. Then ΔABC is congruent to $\Delta A'B'C'$.

We say these triangles are symmetric across L *and that* L *is the line of symmetry.*

PROOF IDEAS: *The plan is to prove the triangles are congruent using SSS. So we first prove a simpler lemma which will show each pair of sides has the same length.*

LEMMA: If L *is the perpendicular bisector of* $\overline{XX'}$ *and* $\overline{YY'}$, *then the lengths* $XY = X'Y'$. *(Draw this on graph paper). We can refer to this lemma as "line reflections preserve lengths".*

In order to prove the lemma, add in the midpoint P *of* $\overline{XX'}$ *and the midpoint* Q *of* $\overline{YY'}$. *Note that* $\overleftrightarrow{PQ}$ *is the line* L. *Use SAS to prove* $\Delta X'QP$ *is congruent to* ΔXQP. *Thus* $m(\angle X'QP) = m(\angle XQP)$. *Mark everything you have so far on your diagram. We still need to show* $XY = X'Y'$ *and we can do this if we can show* $\Delta X'QY$ *is congruent to* ΔXQY. *Do a little work to show that then we have* $m(\angle X'QY) = m(\angle XQY)$ *and then apply SAS to finish the proof of the lemma.*

Complete the proof of the Line of Symmetry Theorem in four lines by applying the lemma three times and then applying SSS.

POINT SYMMETRY: *Plot points* $A = (2, 3)$ $A' = (-2, -3)$ $B = (5, 0)$, $B' = (-5, 0)$, $C = (4, -2)$ *and* $C' = (-4, 2)$ *and observe that triangles* ΔABC *and* $\Delta A'B'C'$ *are "symmetric" with respect to the origin. Note that the origin is the midpoint of* $\overline{AA'}$ *and of* $\overline{BB'}$ *and of* $\overline{CC'}$.

POINT OF SYMMETRY THEOREM: Let P be a point and let ΔABC and $\Delta A'B'C'$ be triangles such that P is the midpoint of $\overline{AA'}$, of $\overline{BB'}$ and of $\overline{CC'}$. Then ΔABC is congruent to $\Delta A'B'C'$.

We say these triangles are symmetric through the point P *and* P *is the point of symmetry.*

PROOF IDEAS: *Carefully draw the given part of the theorem by first drawing a point and one triangle and then using a ruler to reflect each vertex through the point. Then mark lengths which correspond by the definition of midpoint and mark any angles which correspond by the vertical angles theorem. You should find three pairs of congruent triangles. Start the two column proof.*

Now on a second copy of the diagram, mark every side and angle that correspond to each other using the corresponding sides and angles of the three pairs of congruent triangles. With this new information, you should be able to see why ΔABC *is congruent to* $\Delta A'B'C'$ *and write down your final line of the proof using either SAS, SSS or ASA. Complete the two column proof.*