Quantifiers

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A supplement for MAT175

Quantifiers are special symbols that are used to make it easier to think about mathematical statements. There are three quantifiers:

$\forall$ means "for all" "for every"

$\exists$ means "there exists" "there is a"

$\exists!$ means "there exists unique" or "there is only one"

The following statements are true. Why?

$$\forall x \in [0,1],\ sin(x) \text{ is well defined .}$$

and

$$\exists x \in [-1,1] \text{ such that } 1/x \text{ is undefined.}$$

and

$$f : [0,1] \to [0,3] \text{ has an inverse if } \forall y \in [0,3]\ \exists!\, x \in [0,1] \text{ such that } f(x) = y.$$

Now write true and false next to the following statements after translating them. The answers are below so you can check each answer before continuing.

1) $\exists x \in [0, \pi/2]$ such that $sin(x) = 1$.

2) $\exists x \in [0, \pi/4]$ such that $sin(x) = 1$.

3) $\exists!\, x \in [0, \pi/2]$ such that $sin(x) = 0$.

4) $\exists!\, x \in [0, \pi]$ such that $sin(x) = 0$.

5) $\forall y \in [-1,1]\, \exists x \in [0, \pi/2]$ such that $sin(x) = y$.

6) $\forall y \in [-1,1]\, \exists x \in [0, 2\pi]$ such that $sin(x) = y$.

7) $\forall y \in [-1,1]\, \exists!\, x \in [0, 2\pi]$ such that $sin(x) = y$.

8) $\forall y \in [-1,1]\, \exists!\, x \in [-\pi/2, \pi/2]$ such that $sin(x) = y$.

9) $\forall y \in [0,1]\, \exists!\, x \in [0, \pi/2]$ such that $sin(x) = y$.

10) $\forall y \in [0,1]\, \exists x \in [0, \pi/2]$ such that $sin(x) = y$.

The answers to these questions are:
1) T 4) F 8) T 2) F 5) F 9) T 3) T 6) T 10) T 7) F
Try again replacing sin with cos throughout. The answers are different.