Integration Project II

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This project consists of a series of integration problems. In each problem you will have to set up the integral and find a means of solving it. Some of the applications you have seen before and done for HW. Others are new and include explanations. They will require all the integration techniques you know. Be careful when you choose the technique.
Write down your social security number here:
At various times in this project you will be asked to use a digit from your social security number as a constant in the problem. You can work together, but don't forget that your problems are different.

1. Solve the following differential equation:

$$y' = \frac{(3x+5)y}{x^2+4x+3}$$

   with y(0)= the third digit of your social security number.
   Check that your solution obeys the differential equation.
   What happens when x=-1?

2. A shadow is moving along the x axis between -1 and 1. Its velocity at the point x is $t\sqrt{1-x^2}$ and at time 0 the shadow is at 1.
   a)Write this statement down as a differential equation.
   b)Solve the equation for t. Remember that x is a function of t. Now integrate both sides from 0 to T and solve both integrals. Don't forget to change your bounds when you use substitutions.
   c)Where is the shadow at time t?
   d)If you have a particle moving in a circle of radius 1 its path can be described by $x = cos\theta$ and $y = sin\theta$. θ can depend on time. The shadow of such a particle on the x axis would be $x = cos(\theta(t))$. Substitute this formula into the equation you wrote in step a. Differentiate x using the chain rule and find a formula for $\theta'(t)$. Take $\theta(0) = 0$ since x(0)=1 and find the formula for $\theta(t)$. Substitute this formula to get a formula for x and compare with your answer to part c).

3. Integrate the following:

$$\int \frac{2x^3 - x^2 - 13}{x^4 - 18x^2 + 32x - 15}\, dx$$

Hint: Substitute 1 into the denominator to show that it is a root and then use long division to find the rest. Don't forget the $0x^3$.

4. A horse walks on xy plane along the path of a circle of radius 4 about the origin. He pulls a cart which weighs _____N. (Fill in the blank with the last three digits of your social security number). The coefficient of friction at the point (x,y) is y^3x^2.
   a) What is the Force being used to pull the cart at a point (x,y)?
   b) Let θ be the angle in the diagram. A tiny piece of angle is $\Delta\theta$. What is the corresponding piece of distance?
   c) In terms of θ, what are the values of x and y?
   d) What is the equation for the work done while the horse moves a small piece of distance in terms of θ?
   e) What are the range of values for θ if the horse goes halfway around the circle where y is positive?
   f) Compute the amount of work required to move the cart around the halfcircle.

5. a) Draw the hyperbola $x^2 - 3y^2 = 1$ and the lines x=2 and x=-2.
   Warning: when x=1 what is y? Can x ever be 0?
   b) Find and draw the area bounded by the hyperbola and x=2.
   c) Find and draw the area between the other half of the hyperbola and x=-2.
   d) Find and draw the area between the same hyperbola, y=1 and y=-1.
   e) What is the sum of these areas? Did you get 2? Why?