One hour Practice Exam, MAT172, Fall 2005

Exam I on Wed 9/28 will cover A.1-A.6, 1.1-1.7

Here is a sample of problems up through 1.2

1. a) Plot the line of slope 4 through (1,8).

   The answer is a line which passes through (0,4) and (1,8).

   b) What is its y intercept? 4

   c) Write the point slope formula for this line. $y - 8 = 4(x - 1)$

   d) Solve the formula for y and check the slope and y intercept. $y = 4(x - 1) + 8 = 4x - 4 + 8$ so $y = 4x + 4$ so the slope is 4 and the y intercept is 4.

2. a) Solve $-3x + 6 \geq 0$. subtract 6 on both sides: $-3x \geq -6$, divide by -3 and switch inequality $x \leq -6/-3 = 2$. $x \in (-\infty, 2]$.

   b) Then graph $y = -3x + 6$. This is a line through (0,6) and slopping down -3 to (1,3) then (2,0).

   c) What is the slope? -3

   d) Where is $y \geq 0$? on the left of (2,0) where $x \leq 2$ which matches (a).

3. a) What is $(-\infty, 5] \cap (0, \infty)$? This is everything that is in both sets because $\cap$ is the intersection (0,5].

   b) If $x \leq 5$ and $x > 0$, draw the location of values for x on the real line. This the same as in part a), it is an open circle over 0 closed circle over 5 joined between by a line segment.

   c) If $|x| > 4$, draw the location of values for x on the real line. This says the distance from x to 0 is > 4 so it is $x > 4$ or $x < -4$ and is an open circle over the -4 with an arrow to the left and an open circle over the 4 with an arrow to the right. THIS WAS ORIGINALLY POSTED WRONG!

   d) If $|x - 3| > 1$, draw the location of values for x on the real line. This says the distance from x to 3 is more than 1. So $x - 3 > 1$ or $x - 3 < -1$, so $x > 1 + 3 = 4$ or $x < -1 + 3 = 2$ so $x \in (4, \infty) \cup (-\infty, 2)$. This $\cup$ is the union, so anything in either interval works, so open circle above 2 and an arrow to the left is drawn as well as an open circle above 4 and an arrow to the right.

4. Factor $x^2 - 6x + 8$ and then solve $x^2 - 6x + 8 \leq 0$. Write your answer on the number line and in set theory notation.

   $(x-4)(x-2) = x^2 - 4x - 2x + 8 = x^2 - 6x + 8$ works for factoring. This may take a few tries.

   $(x-4)(x-2) \leq 0$ so we have I. (-)(+)=(-) or II. (+)(-)=(-).

   I. $x - 4 \leq 0$ and $x - 2 \geq 0$ OR II. $x - 4 \geq 0$ and $x - 2 \leq 0$.

   working this out we get

   I. $x \in (-\infty, 4] \cap [2, \infty)$ OR II. $x \in [4, \infty) \cap (-\infty, 2]$

   so

   I. $x \in [2, 4]$ OR II. $x \in \emptyset$ because nothing is both bigger than 4 and smaller than 2.

   So $x \in [2, 4]$. Note that 2 and 4 are the roots of the polynomial.

5. Complete the square for $x^2 - 6x + 8 > 0$ and solve $x^2 - 6x + 8 > 0$. Write your answer on the number line and in set theory notation.

   First we wonder $x^2 - 6x + ? = (x + a)^2$, this means $2a = -6$ so $a = -3$.

   So we set up the square to be $(x-3)^2 = x^2 - 6x + 9$.

   So now we go back to $x^2 - 6x + 8 > 0$ and add 1 to be the square

   Which gives us $x^2 - 6x + 9 > 1$ which is $(x-3)^2 > 1$.

   Now this means $|x - 3| > \sqrt{1} = 1$.

   So $x - 3 < -1$ or $x - 3 > 1$. Which is $x < 2$ or $x > 4$.

   In set theory notation we write the OR as a union, $\cup$,

   so we get $x \in (-\infty, 2) \cup (4, \infty)$.

   Notice this is the complement of the set in the last problem because we have just switched the inequality. That is, all real numbers solve the last problem or this problem but no real numbers solve both problems.

6. a) Plot the graph of $(x-3)^2 + (y-2)^2 = 4$.

   This is a circle around (3,2) of radius 2. b) Where does it hit the x axis?

   At (3,0) only. c) Solve for y. $y = \sqrt{4 - (x-3)^2} + 2$ or $-\sqrt{4 - (x-3)^2} + 2$..

   d) Is this a function for y in terms of x? No

   e) Let $y = f(x) = -\sqrt{4 - (x-3)^2} + 2$. Verify this solves the formula in (a).

   f) What is the domain of this function? The radicand must be nonnegative. So solve $4 - (x-3)^2 \geq 0$ and get $4 \geq (x-3)^2$ so $2 \geq |x - 3|$ so $-2 \leq x - 3 \leq 2$, so $1 \leq x \leq 5$. The domain is $[1, 5]$.

e) Plot this function. This is just the bottom semicircle from above. Notice this confirms the domain is [1,5] since those are the only x values under the graph. f) What is the range of the function? looking at the graph we see it is [0,2] since those are all the y values on the semicircle. If we do this algebraically, this is a lot harder.

We know $f(x) \leq 2$ because the negative square root gives something ≤ 0.

To show $f(x) \geq 0$ you need to see how small f can get, which means you need to find the minimum and we will learn how to do that later in the course.

If you were are unable to complete this exam in the one hour then you do not know the material well enough. You need to go to the MATH LAB and practice.

Remember the actual exam covers everything through 1.7!!!!!

The homework is on the class webpage.