Coordinate Geometry Lesson and Project

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Goal: ***To review all the key topics of planar geometry by applying them to the coordinate plane.***

Review: For this project you may use any facts we have discussed this semester from Euclidean geometry and from compass straightedge constructions. You may consult prior projects if you wish. Please write out the statements of theorems and definitions as you apply them.

Key facts from coordinate geometry that will be useful for this project are:

- A line through the point (x_0, y_0) with slope m has the formula $\{y - y_0 = m(x - x_0)\}$. A horizontal line has slope $m = 0$. A vertical line through the point (x_0, y_0) has the formula $\{x = x_0\}$.
- Two lines are parallel iff they have the same slope. Vertical lines are parallel.
- Two lines are perpendicular iff their slopes are negative reciprocals. Vertical lines and horizontal lines are perpendicular.
- The distance between two points (x_0, y_0) and (x_1, y_1) is $\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$.
- The formula for a circle of radius R about a point (x_0, y_0) is
$$\{(x - x_0)^2 + (y - y_0)^2 = R^2\}.$$
- The midpoint between (x_0, y_0) and (x_1, y_1) is $((x_0 + x_1)/2, (y_0 + y_1)/2)$.

Project: Copy the statement of each problem onto the top of a sheet of graph paper and then work it out explicitly writing which definitions and theorems you are using. Be sure to explain and label any formula you write. You may refer to prior problems but a new sheet and a new figure is needed for each problem. When requested to verify a point lies on a line or circle algebraically, one must substitute the x and y coordinates of the point into the formula for the line or circle.

In the following set of problems you will take the following values for the points depending on the number assigned to your group. These points are chosen to have nice answers.

Group I: $A = (2, 4)$, $B = (6, 8)$, $C = (6, 4)$, $r = 4$, $R = 5$, $P = (6, 1)$, $Q = (6, 7)$, $L = \{x = -3\}$
Group II: $A = (8, 3)$, $B = (4, 7)$, $C = (4, 3)$, $r = 4$, $R = 5$, $P = (4, 0)$, $Q = (4, 6)$, $L = \{x = 13\}$.
Group III: $A = (4, 3)$, $B = (8, 7)$, $C = (4, 7)$, $r = 4$, $R = 5$, $P = (1, 7)$, $Q = (7, 7)$, $L = \{y = -2\}$.
Group IV: $A = (1, 8)$, $B = (9, 16)$, $C = (9, 8)$, $r = 8$, $R = 10$, $P = (9, 2)$, $Q = (9, 14)$, $L = \{x = -9\}$.
Group V: $A = (0, 1)$, $B = (8, 9)$, $C = (0, 9)$, $r = 8$, $R = 10$, $P = (-6, 9)$, $Q = (6, 9)$, $L = \{y = -4\}$.

The first seven problems must be done in order as a group. Please be sure to give everyone in the group a little time to think and listen to everyone's suggestions:

1. Find the formula of the line through A and B, verify that the midpoint M of AB lies on this line algebraically, and verify that the distance from the midpoint to A is half the distance from A to B.

2. Find the formula of the perpendicular bisector of AB and graph your answer to verify it is correct and looks perpendicular.

3. Find the formula for the circle of radius r about A and the formula for the circle of radius r about B and graph these circles using a compass. Find the points of intersection of these two circles and verify that both points lie on both circles algebraically. Finally verify that these two points lie on the perpendicular bisector found in the last problem algebraically.

4. Find the formulas of the perpendicular bisectors of each side of ΔABC and graph them. Find the circumcenter D by examining the graph and verify that the circumcenter lies on all three perpendicular bisectors algebraically.

5. Find the circumradius of ΔABC, graph the circumcircle using a compass and find the formula of the circumcircle. Finally verify that all three vertices of ΔABC lie on the circle algebraically.

6. Draw the circumcircle of ΔABC and compute the lengths of the minor arcs AB, BC and CA using $\angle ACB$, $\angle BAC$ and $\angle CBA$.

7. Show that $\angle ABC = \angle ADC/2$, $\angle BAC = \angle BDC/2$ and $\angle CBA = \angle CDA/2$.

   Problems 7-10 may be done individually and completed for homework if there is no time in class.

8. Recall the theorem we proved that the line segment between the midpoints of two sides of a triangle is parallel to the third side of the triangle. Verify this theorem for ΔABC

9. Draw the circle of radius R about A and verify that P lies on this circle. Find the formula for the line through A and P and find the formula of the tangent line to the circle at P. Graph this line and verify it is tangent. Repeat this now using Q in the place of P, then find the point Z where the tangent line at P meets the tangent line at Q algebraically and make sure your graph looks correct. Then verify the tangent theorem: that the lengths $PZ = QZ$.

10. Graph the line through AP and find the formula for the circle of radius R tangent to the line through AP at the point A.

    Problems 11-13 are extra credit problems that build upon one another and upon the above problems:

11. Verify that the circle of radius R about A is tangent to the line, L.

12. Find the point E where L meets the line QD and the point F where L meets the line tangent to PD and explain why the circle of radius R about A is the inscribed circle of ΔEFD.

13. Find the formula of the angle bisector of $\angle FED$.