Metric Spaces: Quiz on Tubular Neighborhoods

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BACKGROUND: This is a quiz on metric spaces, open sets, unions, Euclidean space and discrete space. Students are not expected to know the definition of tubular neighborhood.

DEFN: Given a set $A$ is a metric space, the tubular neighborhood of radius $R > 0$ about $A$ is defined as

$$T_R(A) = \{x : \exists a \in A \text{ such that } d(x, a) < R\}$$

PROBLEM 1: Prove that in any metric space

$$T_R(B_p(r)) = B_p(r + R)$$

PROBLEM 2: Prove that in a discrete metric space,

$$T_R(A) = A \quad \forall R < 1.$$ 

PROBLEM 3: Prove that in Euclidean space, for $r < R$,

$$T_r(\partial B_p(R)) = Ann_p(R - r, R + r).$$

PROBLEM 4: Prove that in any metric space

$$T_r(A) = \bigcup_{a \in A} B_a(r).$$

PROBLEM 5: Prove that in any metric space, for any set $A$, the tubular neighborhood is an open set.