

# MAT 432 and MAT 733 Exam II Sample

Spring 2008 - - - Professor Sormani

## 1. **Proofs** (15 pts)

Prove that the set  $\{(x, y) : y > 2\}$  is an open set using the definition of open. *Undergraduates get full 15 points just for providing an outline of a proof and up to 10 extra credit for completing the proof. Graduate students must prove it to get 15 points.*

2. (35 pts) **CURVES**

Let  $c(t) = (\cos(t/3), \sin(t/3), t)$  with domain  $t = [0, 3\pi]$  be a curve.

(a) Sketch  $c$  indicating  $c(0)$ ,  $c(\pi)$ ,  $c(2\pi)$  and  $c(3\pi)$  above.

(b) Compute  $c'(t)$

(c) Compute  $|c'(t)|$ .

(d) Find the length of  $c$  from  $(1, 0, 0)$  to  $(0, 1, 3\pi/2)$ .

(e) Compute  $T$  at  $t = 0$ .

(f) Compute  $N$  at  $t = 0$ .

(g) Verify  $N$  is perpendicular to  $T$  and then compute  $B$  at  $t = 0$ .

3. **SURFACES** (25 points)

Let  $X : (s, t) = (s, t, s^2 - t^2)$  with domain  $U = B_{(0,0)}(2)$ .

- a) Sketch the surface  $X(U)$  above being sure to indicate the size of units on your axis. Does it pass through  $(0, 0, 0)$ ,  $(1, 1, 0)$ ,  $(0, 1, -1)$  and  $(1, 0, 1)$ ?
- b) Compute  $DX$  and the two special tangent vectors  $X_1$  and  $X_2$ .
- c) Verify  $X$  is a smooth patch.
- d) Find the tangent vectors at  $(1, 1, 0)$ .
- e) Find the normal vector at  $(1, 1, 0)$ .

4. **Metrics and Areas** (25 points)

Let  $X(s, t) = (s, t, 3 - 3s - 3t)$  be a chart with a triangular domain

$U = \{(s, t) : s > 0, t > 0, s + t < 1\}$ . Note that  $X(U)$  is a triangular part of a plane with corners at  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 3)$ .

- a) Find  $DX$  and the tangent vectors  $X_1$  and  $X_2$ . Verify these are correct before continuing by asking the professor.
- b) Compute  $g_{11}$ ,  $g_{12}$ ,  $g_{21}$  and  $g_{22}$ . The solutions are real numbers in this case.
- c) Find  $\det[g_{ij}]$ .
- d) Find the area of  $X(U)$  using integration and  $g_{ij}$ . Be sure to work out the integral until you get a real number.