MAT 432 and MAT 733 Exam I Sample

Professor Sormani

1. (20 points) Let $A = [-2, 1] \times [-5, -3]$. Let $B = B_{(3,4)}(5)$.
   Let $C = \{(x, y) : x^2 + y^2 \geq 100\}$ and let $D = \{(x, y) : y = x - 5\}$.

   a) Draw all of these sets on one chart shading them lightly if they have area. Use dashed lines if they don't include their edges. Use solid lines otherwise.

   b) Which of these sets is open?

   c) Which of these sets is closed?

   d) Which of these sets is bounded?

   e) Which of these sets is compact?

2. (20 pts) **SUBSETS**

Let $A = (1,3) \times (-4,2)$. Show $A \subset B_{(0,0)}(5)$ as follows:

a) Draw a picture of the two sets carefully:

b) Let $(x,y) \in A$ then

$$< x < \qquad\qquad \text{and} \qquad\qquad < y <$$

c) So carefully fill in:

$$x^2 < \qquad\qquad \text{and } y^2 <$$

d) So

$$x^2 + y^2 <$$

e) Complete the proof:

3. (20 pts) **Proof Formating**

Write the first few lines of proofs of the following statements:

a) $\forall p \in A \qquad \exists r_p > 0$ such that $B_p(r_p) \subset A$.

1) Let ....

2) Choose ...

3) We claim:

b) $\forall q \notin K \qquad \exists \epsilon_q > 0$ such that $B_q(\epsilon_q) \cap K = \emptyset$

1)

2)

3) We claim:

c) $\forall p \in A \qquad \forall \epsilon > 0 \qquad \exists \delta > 0$ such that $f(B_p(\delta)) \subset B_{f(p)}(\epsilon)$

1)

2)

3)

4) We claim:

d) $\exists R > 0$ and $\exists p \in S^2 = \{(x, y, z) : x^2 + y^2 + z^2 = 1\}$ such that $S^2 \subset B_p(R)$.

1)

2)

3) We claim

e) Which of the above is:

A proof of continuity:

A proof that a set is open:

A proof that a set is bounded:

A proof that a set is closed:

4. (20 pts) **INVERSES AND CONTINUITY**

Let $f : [1, 3] \to H$ be the function $f(x) = 4x + 2$.

a) Find the image $H = f([1, 3]) =$

b) Find the preimage $f^{-1}((9, 11)) =$

d) Find the inverse of f:

e) Prove f is continuous.

5. (20 pts) **OPEN SETS**

Fill in the three steps showing that $U = \{(x, y) : \ y > x\}$ *is an open set.*

I) Let $(a, b) \in U$**. So we have the following formula with a and b in it:**

II) Choose $r = r_{(a,b)} = dist((a, b), L)$ **where** L **is the line** $y = x$**.**

Draw a picture with U*,* (a, b) *and* L *and label* r*:*

Later you will compute a value for $r_{(a,b)}$ *using this picture.*

III) We claim that:

After filling in the claim, draw a picture corresponding to this claim below:

Find the formula for $r_{(a,b)}$*:*

Extra Credit: Complete the proof of the claim after finishing the exam: