Research Description

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I began my career in Riemannian Geometry, a field where theorems can be stated in an elegant and easily understood manner depending only on basic notions of topology and geometry. From the start I was intrigued in the Gromov-Hausdorff limits of Riemannian manifolds and I began to study their properties. More recently I have begun to develop new notions of convergence of Riemannian manifolds that can be applied in settings where no Gromov-Hausdorff limits exist. I am particularly interested in applications to General Relativity. Here I state some of the key theorems I've proven and notions I've defined divided into these four subject areas.

I have had some wonderful experiences collaborating, yet most of my research has been conducted alone. As a doctoral student at Courant, Jeff Cheeger was my dissertation advisor. I spent one year as a postdoc at Harvard under Shing-Ting Yau and two years as a postdoc at Johns Hopkins with Joel Spruck and Bill Minicozzi before taking a tenure track job at CUNY. I have coauthored multiple papers with Guofang Wei (UCSB) and Zhongmin Shen (IUPUI) and I've engaged in many a lively debate with Dimitri Burago (PSU). More recently I have coauthored with Krishnan Shankar (Oklahoma) and two junior mathematicians: Stefan Wenger (who was a postdoc at NYU) and Dan Lee (who is tenure track at CUNY).

I enjoy working with doctoral students: sometimes coauthoring with them on a project I've begun earlier and other times suggesting problems and providing guidance. My first student, Michael Munn, graduated from CUNY in 2008 and won an NSF International Postdoc. He is now tenure track at U Missouri Columbia. Pedro Solorzano graduated from Stony Brook in 2011 working with Blaine Lawson and me. He is now a postdoc at UC Riverside. My current doctoral students are Sajjad Lazkian (CUNY), Jorge Basilio (CUNY), and Raquel Perales (Stony Brook). I'm also working with Fanghua Lin's student, Jacobus Portegies (NYU).

1. Riemannian Geometry Theorems

In this section, I describe my theorems concerning *complete noncompact Riemannian manifolds, M^m, with nonnegative Ricci curvature.* These are traditional Riemannian Geometry theorems in the sense that they use basic notions from comparison geometry. Key ingredients in the proofs of these theorems are the Bishop-Gromov Volume Comparison Theorem, the Cheeger-Gromoll Splitting Theorem and the Abresch-Gromoll Excess Theorem, along with techniques involving harmonic functions, barriers and the maximum principle. My first theorem was:

Theorem 1. [S–JDG-98] *If M^m has at most linear volume growth,* $\limsup_{r\to\infty} \frac{vol(B_p)(r)}{r} < \infty$, *then for any ray, $\gamma : [0,\infty) \to M^m$, the ray Busemann function, $B_\gamma(x) := \lim_{R\to\infty} R - d(x,\gamma(R))$ has compact level sets. Furthermore the diameters of the level sets grow at most linearly.*

This is a partial solution to a conjecture in Shing-Tung Yau's 1982 problem list. Zhongmin Shen had proven the conjecture assuming at least Euclidean volume growth in [Sh-Invent-96] using different techniques. No one has been able to improve upon our results in the intervening years, nor provide a counter example to Yau's conjecture.

As a postdoc, I began to study Milnor's 1968 conjecture that any complete noncompact manifold, M, with nonnegative Ricci curvature has a finitely generated fundamental group. I observed that all geodesics entering balls about cut points soon stop minimizing. Using this *Uniform Cut Lemma*, I proved the following theorems:

Theorem 2. [S–JDG-00] *There is an explicit constant, S_m, depending on dimension, such that if M^m has diameter growth $\leq S_m r$ then its fundamental group, $\pi_1(M^m)$, is finitely generated.*

Theorem 3. [S–JDG-00] *If M^m has linear volume growth then $\pi_1(M^m)$ is finitely generated.*

The second theorem is a consequence of the first combined with the results in my thesis. Prior to this theorem, Anderson and Li had proven finiteness of the fundamental group under maximal volume growth conditions using covering arguments and heat kernal estimates respectively [A-Top-90][L-Annals-86]. Wilking has also done work in this direction [W-DGA-00]. Various mathematicians have refined the constant, S_m, by fine tuning my estimates [XWY-CAMS-03][XD-AMSC-06]. The full Milnor conjecture remains unresolved to this day.

While studying closed geodesics, I discovered the following theorem and proved it using the Cheeger-Gromoll Splitting Theorem applied to double covers:

Theorem 4. [S–IJM-01] *If M^m has nonnegative Ricci curvature then either it has the loops to infinity property or it is isometric to a flat normal bundle over a compact totally geodesic submanifold and its double cover is split isometrically.*

The loops to infinity property states that any noncontractible loop is homotopic to a sequence of loops diverging to infinity. Zhongmin Shen and I applied this theorem combined with some techniques from algebraic topology to prove the next theorem (my first coauthored result):

Theorem 5. [ShS-AJM-01] (Joint with Shen) *If M^n has nonnegative Ricci curvature then it either has a trivial codimension one integer homology or it is isometric to a flat normal bundle over a compact totally geodesic submanifold and its double cover is split isometrically.*

Yau proved the codimension one real homology was trivial in 1976. Shen and Itokawa-Kobayashi had worked towards classifying the codimension one integer homology using Morse theory and integral currents respectively [S-Invent-96][IK-AJM-99]. Our proof completes the entire classification using only the loops to infinity property without applying this prior work.

I was invited to write a survey about the topology of these manifolds and wrote one together with Zhongmin Shen, suggesting a possible counter example to the Milnor conecture [ShS-CMA-08]. Since then Gang Liu has proven the Milnor conjecture in dimension three by proving a property similar to the loops to infinity property combined with a theorem of Stallings and the fact that any simply connected three manifold is a sphere (which implies M^3 is irreducible) [L-Invent-12]. There has been very little progress in arbitrary dimensions.

2. Gromov-Hausdorff Convergence

In 1981, Gromov first introduced the Gromov-Hausdorff distance between Riemannian manifolds. He proved sequences of manifolds with uniform lower bounds on their Ricci curvature have subsequences which converge in the Gromov-Hausdorff sense to geodesic metric spaces. More generally, he proved metric spaces which are uniformly compact (uniform numbers of balls of radius ϵ required to cover a ball of radius R), have converging subsequences.

My first theorem in this area appeared in my doctoral dissertation completed under the supervision of Jeff Cheeger. It concerned manifolds with quadratically decaying lower bounds on Ricci curvature. Here I write a simplified statement assuming only nonnegative Ricci:

Theorem 6. [S–CAG-98] *If M^m is a complete Riemannian manifold with nonnegative Ricci curvature and linear volume growth, then asymptotically as $R \to \infty$, regions in the manifold, $B_\gamma^{-1}(R, R+L)$, are Gromov-Hausdorff close to isometric products, $X_R \times (R, R+L)$.*

Corollary 7. [S–CAG-98] *Such manifolds have sublinear diameter growth.*

This theorem was proven using the almost rigidity techniques developed by Cheeger-Colding to prove the corresponding theorem with maximal volume growth [ChCo-Annals-96]. Cheeger-Colding then proceeded to produce their trio of papers on the properties of limits of manifolds with uniform lower bounds on their Ricci curvature [ChCo-JDG-97-00]. Colding's student Menguy proved that the limit spaces could have infinite topological type locally [M-Duke-00]. Guofang Wei and I realized that my Uniform Cut Lemma could be applied to to control the topology of the limit spaces as follows:

Theorem 8. [SW-TAMS-01, SW-TAMS-04] (Joint with Wei) *Gromov-Hausdorff limits of complete manifolds with uniform lower Ricci curvature bounds have universal covering spaces.*

The first version of this theorem only applied to compact limit spaces [SW-TAMS-01]. To prove the theorem we developed a notion called a δ cover, $\tilde{M}^\delta$, which is a covering space whose covering map is an isometry when restricted to balls of radius δ. These δ covers have been of independent interest to metric geometers as a means to determine when certain metric spaces have universal covers. They lead naturally to the following concept:

Definition 9. [SW-JDG-04] (Joint with Wei) *The Covering Spectrum of a compact length space, X, is $CovSpec(X) = \{\delta > 0 : \ \tilde{M}^{\delta} \neq \tilde{M}^{\delta'} \ \forall \delta' > \delta\}$.*

Wei and I proved a number of theorems about the covering spectrum in this joint paper (including its relationship to the fundamental group and the length spectrum as well as its continuity under Gromov-Hausdorff convergence). In [SW-TAMS-10] and a more recent preprint, we studied various notions of covering spectra which can be defined on complete noncompact metric spaces. All our papers provide applications both in Riemannian and metric geometry.

I then completed a solo paper on length spectra and Gromov-Hausdorff limits [S–AiM-07]. One may recall that Colin de Verdiere [V-Comp-73] had proven that the length spectrum is determined by the Laplace spectrum on a generic compact Riemannian manifold. His proof involved the heat kernal on the manifold. Duistermaat-Guillemin [DG-Invent-75] reproved his result using their wave trace formula. Although Cheeger-Colding [ChCo-JDG-00] proved the laplace spectrum sometimes behaves well under metric measure convergence, the length spectra is not well behaved at all. In this paper I proved various subspectra of the length spectra converge when the underlying spaces converge in the Gromov-Hausdorff sense [S-AiM-07].

As in the work with Guofang Wei, many of the results can be stated on geodesic spaces without any need for a Riemannian structure. I've completed further investigation into Riemannian-like properties on arbitrary length spaces in a joint paper with Ravi Shankar [ShS-AiM-09]. In particular we extend the notion of a conjugate point and prove Klingenberg's Long Homotopy Lemma [K-AMPA-62] in this nonsmooth setting. There are quite a number of theorems in both of these papers as well as open problem lists.

3. New Notions of Convergence

Stefan Wenger and I introduced the Intrinsic Flat distance between compact oriented Riemannian manifolds in a preprint in 2008 which I presented at the *Geometry Festival* in 2009. It is built upon the work of Ambrosio-Kirchheim which develops the notion of currents on arbitrary metric spaces. We prove that limits obtained under intrinsic flat convergence are integral current spaces: oriented countably $\mathcal{H}^m$ rectifiable metric spaces with integer weight (possibly the 0 space). Just as the Gromov-Hausdorff distance is an intrinsic version of the Hausdorff distance obtained by taking infima over all isometric embeddings into a common space and measuring the Hausdorff distance between them, the intrinsic flat distance is an intrinsic version of the Federer-Fleming flat distance between submanifolds in Euclidean space:

Definition 10. [SW-JDG-11] (Joint with Wenger) *Given two compact oriented Riemannian manifolds, M_1^m, M_2^m, with boundary, we define the intrinsic flat distance:*

$$d_{\mathcal{F}}\left(M_1, M_2\right) := \inf d_F^Z\big(\varphi_{1\#}[M_1], \varphi_{2\#}[M_2]\big), \tag{1}$$

where the infimum is taken over all complete metric spaces (Z, d) and isometric embeddings $\varphi_i : M_i \to Z$ and the flat norm is defined as an infimum over integral currents A^m, B^{m+1} in Z:

$$d_F^Z(T_1, T_2) = \inf \left\{ M(A^m) + M(B^{m+1}) : \ T_1 - T_2 = A + \partial B \right\}. \tag{2}$$

Here B^{m+1} is called the filling and A^m is called the excess boundary.

In [SW-JDG-11] we proved this is a distance in the sense that it is 0 iff there is an orientation preserving isometry between the two oriented manifolds. More generally, the intrinsic flat distance between two integral current spaces is 0 iff there is a current preserving isometry between them (preserving both orientation and weight). We proved the following theorem built upon Gromov's embedding theorem and Ambrosio-Kirchheim's extension of the Federer-Fleming compactness theorem:

Theorem 11. [SW-JDG-11] (Joint with Wenger) *Given a sequence of oriented Riemannian manifolds M_j^m such that $M_j^m \xrightarrow{GH} Y$, then a subsequence converges in the intrinsic flat sense to an integral current space, X, which is either a subset of Y with the restricted metric from Y or the* 0 *integral current space.*

In particular if a sequence of manifolds is collapsing (i.e. their volumes converge to 0), then their intrinsic flat limit is the 0 integral current space. This also occurs when the filling volumes of the manifolds converge to 0.

In [SW-CVPDE], we apply Gromov's notion of filling volume [G-JDG-83] and a method of Greene-Petersen [GP-Duke-92] to prove that the limits agree when the manifolds have uniform linear contractibility functions. As a consequence the GH limits of such spaces are countably $\mathcal{H}^m$ rectifiable. Although sequences of manifolds with nonnegative Ricci curvature are known not to have uniform linear contractibility functions, we applied Cheeger-Colding theory [ChCo-JDG-97] combined with early work of Perelman [P-JAMS-94] to obtain sufficient control on filling volumes to prove the following theorem:

Theorem 12. [SW-CVPDE] (Joint with Wenger) *If a sequence of oriented Riemannian manifolds without boundary, M_j^m has Ricci ≥ 0, diam$(M_j^m) \leq D_0$ and vol$(M_j^m) \geq V_0$ then a subsequence converges in both the intrinsic flat sense and the Gromov-Hausdorff sense to the same limit space.*

Naturally, it is of even more interest to use the intrinsic flat distance to study sequences of manifolds which have no Gromov-Hausdorff limits. According to Wenger's Compactness Theorem, any sequence of oriented Riemannian manifolds that have a uniform upper bound of diameter and volume (and area of the boundary) has a subsequence which converges in the intrinsic flat sense [W-CVPDE-11] Many examples of settings where one can apply intrinsic flat convergence appear in the appendix of our joint paper [SW-JDG-11].

In joint work with my doctoral student, Sajjad Lakzian, we have applied intrinsic flat convergence to study sequences of manifolds which converge smoothly away from singular sets. That is one has (M, g_j) and a set $S \subset M$, such that g_j converges smoothly to g_∞ on $M \setminus S$. One often needs to know whether the metric completion of $(M \setminus S, g_\infty)$ agrees with the GH limit.

Theorem 13. [LS-CAG-12] (Joint with Lakzian) *Let (M, g_i) be a sequence of compact oriented Riemannian manifolds such that there is a closed submanifold, S, of codimension two where g_i converge smoothly to g_∞ on $M \setminus S$. If there exists a connected precompact exhaustion, W_j, of $M \setminus S$ satisfying*

$$diam_{g_i}(W_j) \le D_0 \;\; \forall i \ge j, \qquad vol_{g_i}(\partial W_j) \le A_0, \tag{3}$$

$$\text{and } vol_{g_i}(M \setminus W_j) \le V_j \text{ where } \lim_{j\to\infty} V_j = 0. \tag{4}$$

Then $\lim_{i\to\infty} d_{\mathcal{F}}(M_i', N') = 0$ *where N' is the settled completion of $(M \setminus S, g_\infty)$.*

As an immediate consequence manifolds with nonnegative Ricci curvature (or uniform linear contractibiliy functions) that satisfy the hypothesis of this theorem converge in the Gromov-Hausdorff sense to the metric completion of $(M \setminus S, g_\infty)$. In fact we are able to obtain the same result assuming only a uniform lower bound on Ricci curvature:

Theorem 14. [LS-CAG-12] (Joint with Lakzian) *Let (M, g_i) be a sequence of oriented compact Riemannian manifolds with uniform lower Ricci curvature bounds, such that there is a closed submanifold, S, of codimension two where g_i converge smoothly to g_∞ on $M \setminus S$. If there exists a connected precompact exhaustion, W_j, of $M \setminus S$ satisfying (3) and (4) then* $\lim_{i\to\infty} d_{GH}(M_i, N) = 0$, *where N is the metric completion of $(M \setminus S, g_\infty)$.*

We have examples demonstrating the necessity of these hypotheses except for the codimension condition on S. Indeed, in his doctoral thesis, Sajjad Lakzian has managed to extend these results by requiring only that $\mathcal{H}_{m-1}(S) = 0$.

Naturally one needs to know what quantities are conserved and continuous under intrinsic flat convergence. The key properties Federer-Fleming desired when they first defined the flat distance were properties needed to prove the Plateau problem: a strong notion of boundary that persisted under convergence and a strong notion of area that was at least lower semicontinuous [FF-Annals-60]. Ambrosio-Kirchheim proved that integral currents in metric spaces, share these properties [AK-Acta-00]. Wenger and I proved that any sequence of manifolds converging in the intrinsic flat sense can be isometrically embedded into a common space along with their limit in [SW-JDG-11]. This led to:

Theorem 15. [SW-JDG-11] (Joint with Wenger) *If M_j are compact Riemannian manifolds and $\lim_{j\to\infty} d_{\mathcal{F}}(M_j, M_\infty) = 0$, then we have:*

$$\lim_{j\to\infty} d_{\mathcal{F}}(\partial M_j, \partial M_\infty) = 0, \tag{5}$$

$$\liminf_{j\to\infty} vol(M_j) \geq \mathbf{M}(M_\infty), \tag{6}$$

$$\lim_{j\to\infty} FillVol(\partial M_j) = FillVol(\partial M_\infty), \tag{7}$$

where $\mathbf{M}$ is the Ambrosio-Kirchheim mass of the integral current space. It is just the Hausdorff measure when the tangent spaces are Euclidean and otherwise needs an area factor.

The convergence of the boundaries is simple and immediate. The difficult part of the proof of the rest of this theorem was to construct a common separable metric space into which we could isometrically embed the entire sequence and its limit. This space is one dimension higher than the sequence and is in fact countably $\mathcal{H}^{m+1}$ rectifiable. Unlike Gromov's Embedding Theorem, we cannot create a space simply by taking disjoint unions of the sequence itself and finding a clever common metric. We need room for the fillings to estimate the flat distances within this new space and obtain weak convergence there. Note that our common space is not compact. The Ilmanen example of a sequence of spheres with increasingly many increasingly thin splines does not isometrically embed into a compact metric space, although I have shown this sequence converges to the standard sphere in the appendix to [SW-JDG-11].

In [S–ArXiV], I have a theorem which addresses the disappearance of regions in the limit

Theorem 16. [S–ArXiV] *If a sequence of compact oriented Riemannian manifolds, M_i^m, has $M_i^m \xrightarrow{\mathcal{F}} M_\infty^m$ where M_∞^m is a nonzero precompact integral current space then there exists compact submanifolds, $N_i^m \subset M_i^m$ such that $N_i^m \xrightarrow{GH} M_\infty^m$ and $\liminf_{i\to\infty} vol(N_i^m) \geq \mathbf{M}(M_\infty^m)$. If, in addition, $vol(M_i^m) \to \mathbf{M}(M_\infty^m)$ then $vol(M_i^m \setminus N_i^m) \to 0$.*

The fact that points can disappear in the limit makes it impossible to prove a simple Arzela-Ascoli Theorem where the target spaces converge in the flat sense. It also makes it difficult to prove a Bolzano-Weierstrass Theorem for sequences of points in the space. Nevertheless I have proven a variety of Arzela-Ascoli type theorems under additional conditions as well as a Bolzano-Weierstrass type theorem for nondisappearing points and I am adding these theorems to [S–ArXiV] as they are proven. These theorems will be useful to a project proposed by Gromov in [G-CEJM-12] where he suggests applying intrinsic flat convergence to measure the stability of the Burago-Ivanov Volume Rigidity Theorem [BI-GAFA-94] [BI-GAFA-95].

It is also of interest to understand what happens to the level sets of distance functions under intrinsic flat convergence. Applying the Ambrosio-Kirchheim extension [AK-Acta-00] of the

Federer-Fleming Slicing Theorem [FF-Annals-60], I have proven the level sets of distance functions from converging sequences of points convergence. Combining this with the coarea formula, I can use estimates on the filling volumes of level sets of prevent the disappearance of points in the limit. That is, I define:

Definition 17. [S–ArXiV] *Given points $q_1, ..., q_k \in M^m$, where $k < m$, with distance functions $\rho_i(x) = d(x, q_i)$, we define the sliced filling volume of a sphere $\partial B(p, r)$ to be*

$$\mathbf{SF}(p, r, q_1, ..., q_k) = \int_{t_1=m_1}^{M_1} \int_{t_2=m_2}^{M_2} \cdots \int_{t_k=m_k}^{M_k} \mathrm{FillVol}(\partial\,\mathrm{Slice}(B(p, r), \rho_1, ...\rho_k, t_1, ..., t_k))\,\mathcal{L}^k \tag{8}$$

where $m_i = \min\{\rho_i(x) : x \in \bar{B}_p(r)\}$ and $M_i = \max\{\rho_i(x) : x \in \bar{B}_p(r)\}$ and where the slice is defined as in Geometric Measure Theory so that it is supported on $B(p, r) \cap \rho_1^{-1}(t_1) \cap \cdots \cap \rho_k^{-1}(t_k)$.

I use lower bounds on this sliced filling volume and an integral sliced filling volume to prove compactness theorems in which no points disappear in the limit (so the intrinsic flat and Gromov-Hausdorff limits agree). It is perhaps easiest to simply state a three dimensional compactness theorem I've proven using the $k = 2$ sliced filling volume. Keep in mind that there are versions in every dimension and also integral statements as well:

Theorem 18. [S–ArXiV] *Given $r_0 > 0, \beta \in (0, 1), C > 0, V_0 > 0$, If a sequence of Riemannian manifolds, M_i^3, has $vol(M_j^3) \le V_0$, $diam(M_i^3) \le D_0$ and the C, β tetrahedral property for all balls, $B_p(r) \subset M_i^3$, of radius $r \le r_0$:*

$$\exists p_1, p_2 \in \partial B_p(r) \text{ such that } \forall t_1, t_2 \in [(1-\beta)r, (1+\beta)r] \text{ we have}$$

$$\inf\{d(x, y) : x \ne y,\ x, y \in \partial B_p(r) \cap \partial B_{p_1}(t_1) \cap \partial B_{p_2}(t_2)\} \in [Cr, \infty)$$

then a subsequence of the M_i converges in the Gromov-Hausdorff sense. See Figure 1.

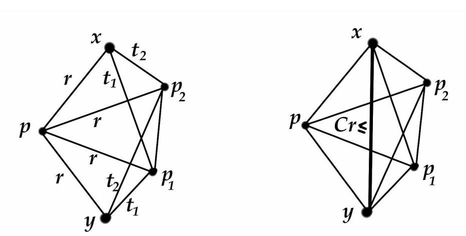

FIGURE 1. Three dimensional tetrahedral property [S–ArXiV]

I have also been working on developing other notions of convergence. I am devising a notion of varifold spaces and a corresponding convergence. Unlike currents, varifolds are not oriented, so their limits would not have the disappearance of regions due to cancellation. With Burago and Ivanov, we have discussed a possible notion of area convergence where the limits are not even metric spaces. They completed the first step in this direction in [BI-GAFA-09].

4. Applications to General Relativity

In General Relativity, the spacelike universe is a Riemannian manifold. There are various assumptions made on the spacelike universe by cosmologists to make predictions about the timing the Big Bang and the expansion of universe. Naturally their models are only models, and reality can only approximate their models up to a certain error. So one needs to understand the error or distance between the real spacelike universe and manifold determined by the model.

In [S–GAFA-04] I studied the Friedmann model of cosmology. In that model the spacelike universe is observed to be isotropic (the same when viewed in all directions at any point in the universe). The conclusion by Schur's Lemma is that the universe has constant sectional curvature. Naturally mass is not distributed uniformly and geodesics can undergo both strong gravitational lensing or weak gravitational lensing, and so the curvatures vary as one looks in different directions. Gribcov and Currier proved Schur's lemma is not stable under smooth perturbations of the metric [G-MS-83][C-PAMS-90]. It is easy to construct natural models of the universe (e.g. the Dyer-Roeder Swiss cheese models [DR-Astro-73]) in which the spacelike universe is not smoothly close to a space of constant sectional curvature.

I proved that: *under conditions which allow for both weak and isolated strong gravitational lensing, one has a Riemannian manifold which is Gromov-Hausdorff close to a collection of spaces of constant sectional curvature glued together at points.* It is easy to construct examples which are close to pairs of spheres of different radii, for example, just by placing a Gromov-Lawson black hole tunnel between them. *If I do not allow localized strong gravitational lensing, only allowing weak gravitational lensing, then the space is Gromov-Hausdorff close to a single space of constant sectional curvature.* The proof of this theorem involves an application of Gromov's Compactness Theorem and the corresponding Arzela-Ascoli Theorem. The limit space is then proven to have so much symmetry its universal cover must be Euclidean space, Hyperbolic space or the Sphere. [S–GAFA-04]

There are many settings arising naturally in General Relativity where the spacelike universe does not have the properties needed to apply Gromov's Compactness Theorem. The positive energy condition allows us to say that a time symmetric spacelike sheet has nonnegative scalar curvature, but one does not have nonnegative Riemannian Ricci curvature even in a vacuum. One of the reasons I first looked into a possible Intrinsic Flat distance was to address these questions.

Recall that the Schoen-Yau Positive Mass Theorem states that an asymptotically flat Riemannian manifold, M^3, with nonnegative scalar curvature has nonnegative ADM mass. If the ADM mass is 0, then M^3 is Euclidean space [SY-CMP-79]. The Penrose Inequality states that if M^3 has an outward minimizing boundary, then $m_{ADM}(M^3) \geq m_H(\partial M^3)$. This was proven by Huisken-Ilmanen and Bray [HI-JDG-01] [B-JDG-01]. One may naturally ask: *what happens when a sequence of such M_j^3 has* $\lim_{j\to\infty} m_{ADM}(M_j) = 0$? Examples demonstrate that such sequences need not converge in the smooth or Gromov-Hausdorff sense to Euclidean space (see the last two columns in Figure 2).

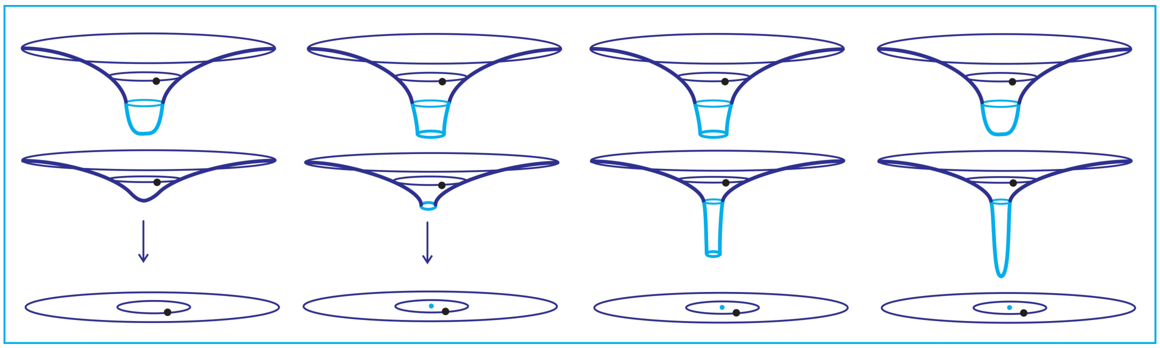

FIGURE 2. Sequences with ADM mass converging to 0 [LS-Crelle].

However, Dan Lee and I have conjectured that, choosing an appropriate selection of basepoints (as in Figure 2), *these spaces converge in the pointed intrinsic flat sense to Euclidean space*. We've proven this in the rotationally symmetric setting in our first preprint together [LS-Crelle]. We also have a preprint concerning almost equality in the Penrose Inequality [LS-Poincare].

I am currently working with Lars Andersson and Ralph Howard on a project suggested by Shing-Tung Yau in which we will extend the notion of intrinsic flat convergence to Lorentzian manifolds. Keep in mind that the flat distance between manifolds is like a Lebesgue distance or an energy. The Gromov-Hausdorff distance is controlled by a maximum principal and the elliptic equations related to nonnegative Ricci curvature on Riemannian manifolds. It can also be useful for studying parabolic partial differential equations like Ricci flow. However, one cannot control the Gromov-Hausdorff distances under the action of a hyperbolic partial differential equation like the Einstein equation on Lorentzian manifolds. So one naturally expects the intrinsic flat distance to be more useful in this setting.

5. FAMILY

I am indebted to my collaborators Guofang Wei, Zhongmin Shen and Dimitri Burago, who all had children young, as I did, before tenure. They showed me how to maintain a research career despite the high teaching loads and service responsibilities associated with working at a public university. I am deeply thankful to my husband, my parents and my in-laws for all their help, and to my three children for their independence, responsibility and loving support.