$with(combinat);$

*Answers first practice* **for** *midterm*

1

$\frac{5!}{2!\cdot 2!}$; $numbcomb(9, 5)$; $2^8$; $15\cdot 14\cdot 13\cdot 12$; $numbcomb(10, 5)$;
$numbcomb(10, 5)$;*#numbcomb(a,b) is a choose b*

2 a. $AD^cBC \cap ADBC = AD^cDBABC = A$
$\varnothing BABC$ (*BY COMMUTATIVITY AND ASSOCIATIVITY OF INTERSECTION*) $= \varnothing$ (*AS* $\varnothing$ *IS IN INTERSECTION*)

2 b. $ABC \cup ABC^c = AB(C \cup C^c) = AB$ **by** *distributive* **and** $C \cup C^c = S$;
*Similarly* $\boldsymbol{AB^cC \cup AB^cC^c = AB^c}$ **and** $\boldsymbol{A^cBC \cup A^cBC^c}$
$\boldsymbol{= A^cB}$ **and** $\boldsymbol{A^cB^cC \cup A^cB^cC = A^cB^c}$**;**

*Now* $\boldsymbol{AB \cup AB^c = A(B \cup B^c)} = A$ **and** *similarly* $\boldsymbol{A^cB \cup A^cB^c = A^c}$

*Finally* $\boldsymbol{A \cup A^c = S}$

3. *The probability you get different colors is the event RB* **or** *BR* (*order matters -- is less prone* **to** *mistakes*, *can also* **use** *easy conditional here*);
*probability of RB is* $\left(\frac{2}{10}\right)\cdot\left(\frac{2\cdot}{10}\right)$; *probability of BR is* $\left(\frac{8}{10}\right)\cdot\left(\frac{8}{10}\right)$
. *So just add.*

4. *Let ET be the event smoking cigarettes* **and** *AR the event smoking cigars*. *Then we have* $P(ET) = .28$; $P(AR) = .07$; $P(ET \cap AR) = .05$. *Then* $P(ET \cup AR) = .28 + .07 - .05 = .3.$

4 a. *we have the event* $(ET \cup AR)^c$. *The probability is* $1 - P\left(ET \bigcup AR\right)$
$= .7$

4 b.*We are looking* **for** $ET \cap AR^c$. *But* $\left(ET \bigcap AR^c\right) \cup \left(ET \bigcap AR\right) = ET$

**and** *this is a partition*. *So* $P\left(ET \bigcap AR^c\right) = P(ET) - P\left(ET \bigcap AR\right)$
$= .28 - .05 = .23.$

5. $P(B1) = .6$; $P(B2) = .5$; $P(B1B2)$
$= .4$ *where these events are the probabilities of liking the respective books*

. *We are asked* **for** $P\left(B2 \middle| B1^c\right) = \frac{P\left(B2B1^c\right)}{P\left(B1^c\right)}$; *Only tricky thing is* **to** *find* $P\left(B2B1^c\right) = P(B2) - P(B2B1)$. *Put* **in** *numbers*.

6. $P(AB) = P(A)P(B)$ *is given*. $P\left(A \middle| B\right) = \frac{P(AB)}{P(B)} = \frac{P(A) \cdot P(B)}{P(B)}$

$= P(A)$ (*which is an important result* **in** *itself* **for** *independence*) *Now* $P\left(A \middle| B^c\right) = \frac{P\left(AB^c\right)}{P\left(B^c\right)} = \frac{P\left(AB^c\right)}{1 - P(B)}$; *But* $P\left(AB^c\right) = P(A) - P(AB) = P(A) - P(A) \cdot P(B) = P(A) \cdot (1 - P(B))$ **and** *we get* $P\left(A \middle| B^c\right) = \frac{P(A) \cdot (1 - P(B))}{1 - P(B)} = P(A)$ **and** *the question is answered*

. *Note that these are theorems that I had gone over* **in** *class* **and** *will go over with you before the test.*

7. *a*. $P(T2) = P(T2H1) + P\left(T2H1^c\right) = P(T2H1) + P(T2T1) = P\left(T2 \middle| H1\right) \cdot P(H1) + P\left(T2 \middle| T1\right) \cdot P(T1)$; *Now* $P\left(T2 \middle| H1\right) = 1 - P\left(H2 \middle| H1\right) = .3$ **and** *we know all other values* **from** *problem*.

7. *b*. $P\left(T1 \middle| T2\right) = \frac{P(T1T2)}{P(T2)}$; *We know* $P(T2)$ **from** *a* **and** $P(T1T2) = P\left(T2 \middle| T1\right) \cdot P(T1)$ *which we know*. *So we are* done.