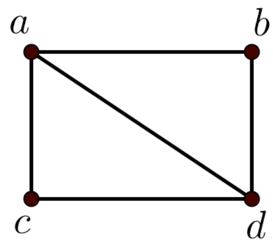
MAT 237/CMP 232: Problem Sheet #6

Topic: Representing Graphs, Graph Isomorphisms, and Graph Connectivity

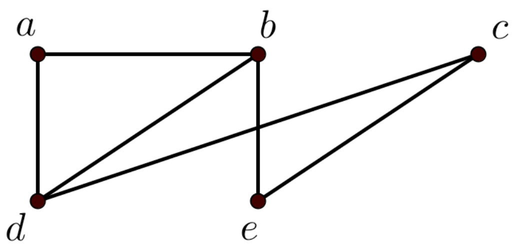
Instructions. The following is a collection of questions pertaining to the topic indicated above. Please bring this worksheet to class for each day we discuss this topic. Though some problems will be assigned to solve for homework, others will be discussed in class.

Problems

1. Use an adjacency list to represent the given graphs.



(a)



(b)

2. Represent the graph in (a) above with an adjacency matrix.

3. Represent the graph in (b) above with an adjacency matrix.

4. Represent each of the following graphs with an adjacency matrix.

   (a) K_4 (c) C_4

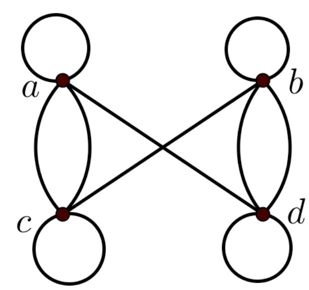
   (b) $K_{1,4}$ (d) W_4

5. For each matrix below, draw a graph with that adjacency matrix (if possible).

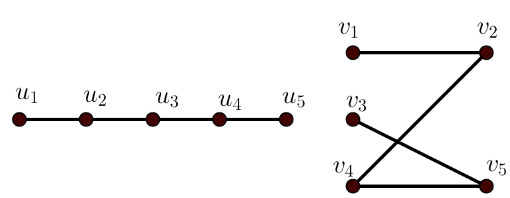
   (a) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 3 & 2 \\ 3 & 0 & 4 \\ 2 & 4 & 0 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$

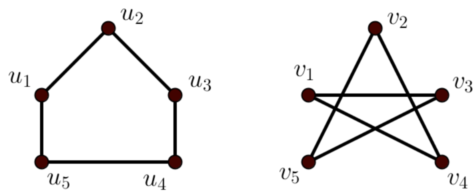
6. Represent the given graph using an adjacency matrix.



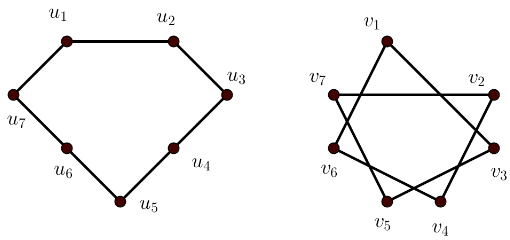
7. Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.



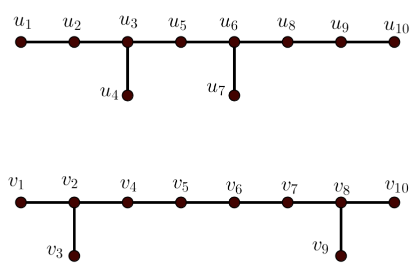
(a)



(b)



(c)



(d)

8. Are the simple graphs with the following adjacency matrices isomorphic?

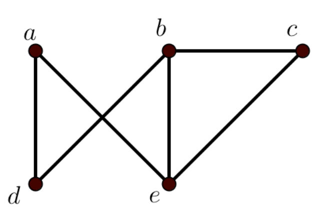
(a) $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$

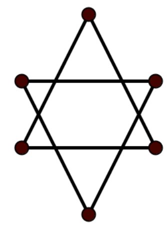
9. Do each of these lists of vertices form a path in the graph below? Which paths are simple? Which are circuits? What are the lengths of those that are paths?

(a) a, e, b, c, b (c) e, b, a, d, b, e

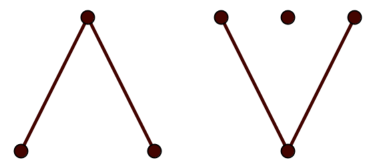
(b) a, e, a, d, b, c, a (d) c, b, d, a, e, c



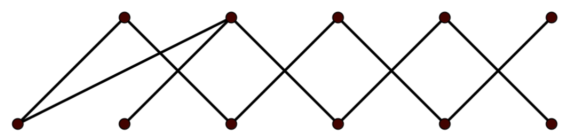
10. Determine whether the given graph is connected.



(a)



(b)



(c)

11. Find the number of paths of length n between two different vertices in K_4 if n is:

(a) 2 (b) 3 (c) 4 (d) 5

12. Use paths to either show that these graphs are not isomorphic or find an isomorphism between them.