MAT 237/CMP 232: Problem Sheet #5

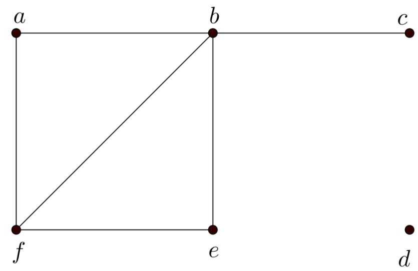
Topic: The Binomial Theorem, Function Growth, and Introduction to Graphs

Instructions. The following is a collection of questions pertaining to the topic indicated above. Please bring this worksheet to class for each day we discuss this topic. Though some problems will be assigned to solve for homework, others will be discussed in class.

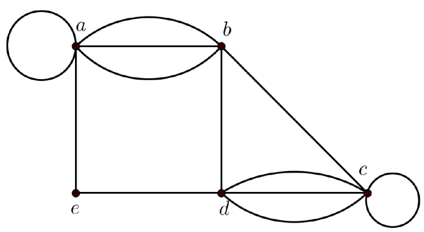
Problems

1. Find the expansion of $(x+y)^5$
2. Find the coefficient of x^5y^8 in $(x+y)^{13}$.
3. What is the coefficient of x^7 in $(1+x)^{11}$.
4. What is the coefficient of x^8y^9 in the expansion of $(3x+2y)^{17}$.
5. Explain why $\binom{n}{k} \leq 2^n$ for all positive integers n and all positive integers k with $0 \leq k \leq n$.
6. Prove that if n and k are integers that satisfy $1 \leq k \leq n$, then $k\binom{n}{k} = n\binom{n-1}{k-1}$. (*It may help to think about choosing a committee and a committee leader...*)
7. Determine whether each of these functions is $O(x)$.
   (a) $f(x) = 10$
   (b) $f(x) = 3x + 7$
   (c) $f(x) = x^2 + x + 1$
   (d) $f(x) = 5\log(x)$
   (e) $f(x) = \lfloor x \rfloor$
   (f) $f(x) = \lceil x/2 \rceil$
8. Determine whether each of these functions is $O(x^2)$.
   (a) $f(x) = 17x + 11$
   (b) $f(x) = x^2 + 1000$
   (c) $f(x) = x\log(x)$
   (d) $f(x) = x^4/2$
   (e) $f(x) = 2^x$
   (f) $f(x) = \lfloor x \rfloor \cdot \lfloor x \rfloor$
9. Show that $(x^2+1)/(x+1)$ is $O(x)$.
10. Show that $(x^3+2x)/(2x+1)$ is $O(x^2)$
11. Let k be a positive integer. Show that $1^k + 2^k + \cdots + n^k$ is $O(n^{k+1})$.
12. The **intersection graph** of a collection of sets $A_1, A_2, \ldots, A_n$ is the graph that has a vertex for each of these sets and has an edge connecting the vertices representing two sets if these sets have a nonempty intersection. Construct the intersection graph of these collections of sets.
    (a) $A_1 = \{0, 2, 4, 6, 8\}$,
    $A_2 = \{0, 1, 2, 3, 4\}$,
    $A_3 = \{1, 3, 5, 7, 9\}$,
    $A_4 = \{5, 6, 7, 8, 9\}$,
    $A_5 = \{0, 1, 8, 9\}$
    (b) $A_1 = \{\ldots, -4, -3, -2, -1, 0\}$,
    $A_2 = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$,
    $A_3 = \{\ldots, -6, -4, -2, 0, 2, 4, 6, \ldots\}$,
    $A_4 = \{\ldots, -5, -3, -1, 1, 3, 5, \ldots\}$,
    $A_5 = \{\ldots, -6, -3, 0, 3, 6, \ldots\}$
    (c) $A_1 = \{x : x < 0\}$,
    $A_2 = \{x : -1 < x < 0\}$,
    $A_3 = \{x : 0 < x < 1\}$,
    $A_4 = \{x : -1 < x < 1\}$,
    $A_5 = \{x : x > -1\}$,
    $A_6 = \mathbb{R}$
13. Can a simple graph exist with 15 vertices each of degree 5?
14. Draw these graphs.
    (a) K_7
    (b) $K_{1,8}$
    (c) $K_{4,4}$
    (d) C_7
    (e) W_7
    (f) Q_4

15. For each of the following, find the number of vertices, the number of edges, and the degree of each vertex. Identify all isolated and pendant vertices.

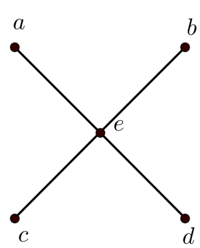


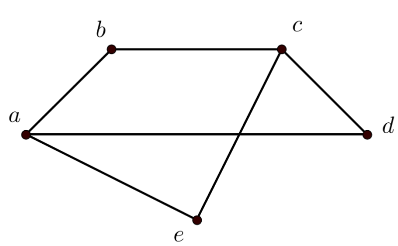
(a)

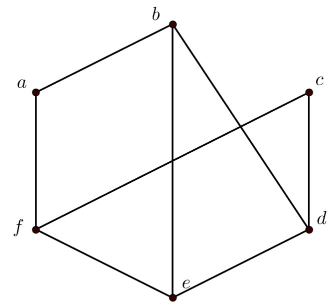


(b)

16. Determine whether the following graph is bipartite. You can answer this question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.



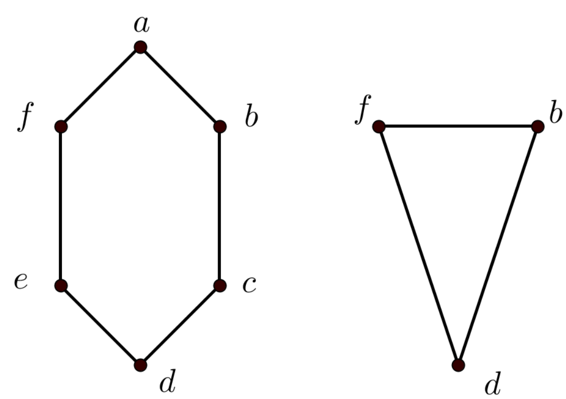




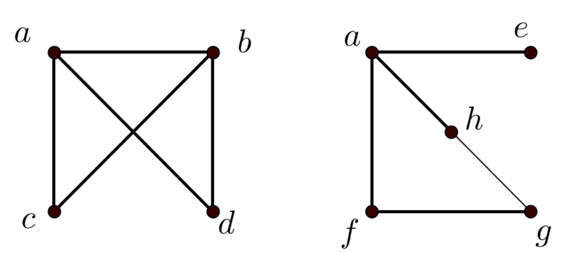
17. How many vertices and how many edges do these graphs have?

(a) K_n (b) C_n (c) W_n (d) $K_{m,n}$ (e) Q_n

18. In the following, find the union of the given pair of simple graphs:



(a)



(b)