MAT 237/CMP 232: Problem Sheet #4

Topic: Counting Techniques and Principles

Instructions. The following is a collection of questions pertaining to the topic indicated above. Please bring this worksheet to class for each day we discuss this topic. Though some problems will be assigned to solve for homework, others will be discussed in class.

Problems

1. An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

2. A multiple-choice text contains 10 questions. There are four possible answers for each question.

   (a) In how many ways can a student answer the questions on the test if the student answers each question?

   (b) In how many ways can a student answer the questions on the test if the student can leave answers blank?

3. A particular brand of shirt comes in 12 colors, has a male and female version, and comes in three sizes for each sex. How many different types of this shirt are made?

4. How many strings are there of lowercase letters of length four or less, not counting the empty string?

5. How many strings are there of four lowercase letters that have the letter x in them?

6. How many strings of three decimal digits

   (a) do not contain the same digit three times?

   (b) begin with an odd digit?

   (c) have exactly two digits that are 4s?

7. A committee is formed consisting of one representative from each of the 50 states in the United States, where the representative from a state is either the governor or one of the two senators from that state. How many ways are there to form the committee?

8. How many license plates can be made using either three digits followed by three uppsercase English letters or three uppercase English letters followed by three digits?

9. How many different functions are there from a set with 10 elements to sets with the following number of elements?

   (a) 2 (c) 4

   (b) 3 (d) 5

10. How many one-to-one functions are there from a set with five elements to sets with the following number of elements?

    (a) 4 (c) 6

    (b) 5 (d) 7

11. Every student in a discrete mathematics class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 38 computer science majors (including joint majors), 23 mathematics majors (including joint majors), and 7 joint majors?

12. Show that if there are 30 students in a class, then at least two of them have last names that begin with the same letter?

13. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

    (a) How many socks must he take out to be sure that he has at least two socks of the same color?

    (b) How many socks must he take out to be sure that he has at least two black socks?

14. What is the minimum number of students, wach of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

15. (a) Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9.

    (b) Is the conclusion in part (a) true if four integers are selected rather than five?

16. List all permutations of $\{a, b, c\}$.

17. Let $S = \{1, 2, 3, 4, 5\}$.

    (a) List all the 3-permutations of S.

    (b) List all the 3-combinations of S.

18. Find the value of each of these quantities.

    (a) $P(6, 3)$
    (b) $P(6, 5)$
    (c) $P(8, 1)$
    (d) $P(8, 5)$
    (e) $P(8, 8)$
    (f) $P(10, 9)$

19. Find the value of each of these quantities.

    (a) $C(5, 1)$
    (b) $C(5, 3)$
    (c) $C(8.4)$
    (d) $C(8, 8)$
    (e) $P(8, 0)$
    (f) $C(12, 6)$

20. In how many different ways can five runners finish a race if no ties are allowed?

21. There are six different candidates running for governor of a state. In how many different orders can the names be printed on the ballot?

22. A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

23. In how many ways can a set of five letters be chosen from the English language?

24. How many subsets with more than two elements does a set with 100 elements have?

25. A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes

    (a) are there in total?

    (b) contain exactly two heads?

    (c) contain atleast three heads?

    (d) contain the same number of heads and tails?

26. A club has 25 members.

    (a) How many ways are there to choose four members of the club to serve on an executive committee?

    (b) How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?