MAT 237/CMP 232: Problem Sheet #1

Topic: Sets and Set Operations

Instructions. The following is a collection of questions pertaining to the topic indicated above. Please bring this worksheet to class for each day we discuss this topic. Though some problems will be assigned to solve for homework, others will be discussed in class.

Problems.

1. List the members of these sets.
   (a) $\{x : x \text{ is a real number such that } x^2 = 1\}$
   (b) $\{x : x \text{ is a positive integer less than } 12\}$
   (c) $\{x : x \text{ is the square of an integer and } x < 100\}$
   (d) $\{x : x \text{ is an integer such that } x^2 = 2\}$
2. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.
   (a) the set of airline flights from New York to New Delhi, the set of nonstop airline flights from New york to New Delhi.
   (b) the set of people who speak English, the set of people who speak Cantonese.
   (c) the set of flying squirels, the set of living creatures that can fly.
3. What is the cardinality of each of these sets?
   (a) $\{a\}$ (b) $\{\{a\}\}$ (c) $\{a, \{a\}\}$ (d) $\{a, \{a\}, \{a, \{a\}\}\}$
4. What is the cardinality of each of these sets?
   (a) $\emptyset$ (b) $\{\emptyset\}$ (c) $\{\emptyset, \{\emptyset\}\}$ (d) $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}\}$
5. Find the power set of each of these sets, where a and b are distinct elements.
   (a) $\{a\}$ (b) $\{a, b\}$ (c) $\{\emptyset, \{\emptyset\}\}$
6. Can you conclude that $A = B$ if A and B are two sets with the same power set? Explain.
7. Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$. Find $A \times B$ and $B \times A$.
8. Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find
   (a) $A \times B \times C$ (b) $C \times B \times A$ (c) $C \times A \times B$ (d) $B \times B \times B$

9. Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets.

   (a) $A \cap B$ (b) $A \cup B$ (c) $A - B$ (d) $B - A$

10. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

    (a) $A \cap B$ (b) $A \cup B$ (c) $A - B$ (d) $B - A$

11. For each of the following, assume that set A is a subset of some underlying universal set U.

    (a) Prove that $\overline{\overline{A}} = A$.
    (b) Prove that $A \cup U = U$.
    (c) Prove that $A \cap \emptyset = \emptyset$.
    (d) Prove that $A \cup \overline{A} = U$.
    (e) Prove that $A \cap \overline{A} = \emptyset$.
    (f) Prove that $A - \emptyset = A$.

12. Let A and B be sets. Show that

    (a) $(A \cap B) \subseteq A$
    (b) $A \subseteq (A \cup B)$
    (c) $A - B \subseteq A$
    (d) $A \cap (B - A) = \emptyset$

13. Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

    (a) $A \cap B \cap C$ (b) $A \cup B \cup C$ (c) $(A \cup B) \cap C$ (d) $(A \cap B) \cup C$

14. The **symmetric difference** of sets A and B is denoted by $A \oplus B$; it is the set containing of elements in either A or B, but not in both. Use this to answer the following questions.

    (a) Let $A = \{1, 3, 5\}$ and $B = \{1, 2, 3\}$. Find $A \oplus B$.
    (b) Describe the symmetric difference of the set of computer science majors at Lehman and the set of mathematics majors at Lehman.
    (c) What can you say about the sets A and B if $A \oplus B = A$?

15. Let $A_i = \{1, 2, 3, ..., i\}$ for $i = 1, 2, 3, ...$ Find

    (a) A_1
    (b) A_2
    (c) A_{16}
    (d) $\bigcup_{i=1}^{n} A_i$
    (e) $\bigcap_{i=1}^{n} A_i$