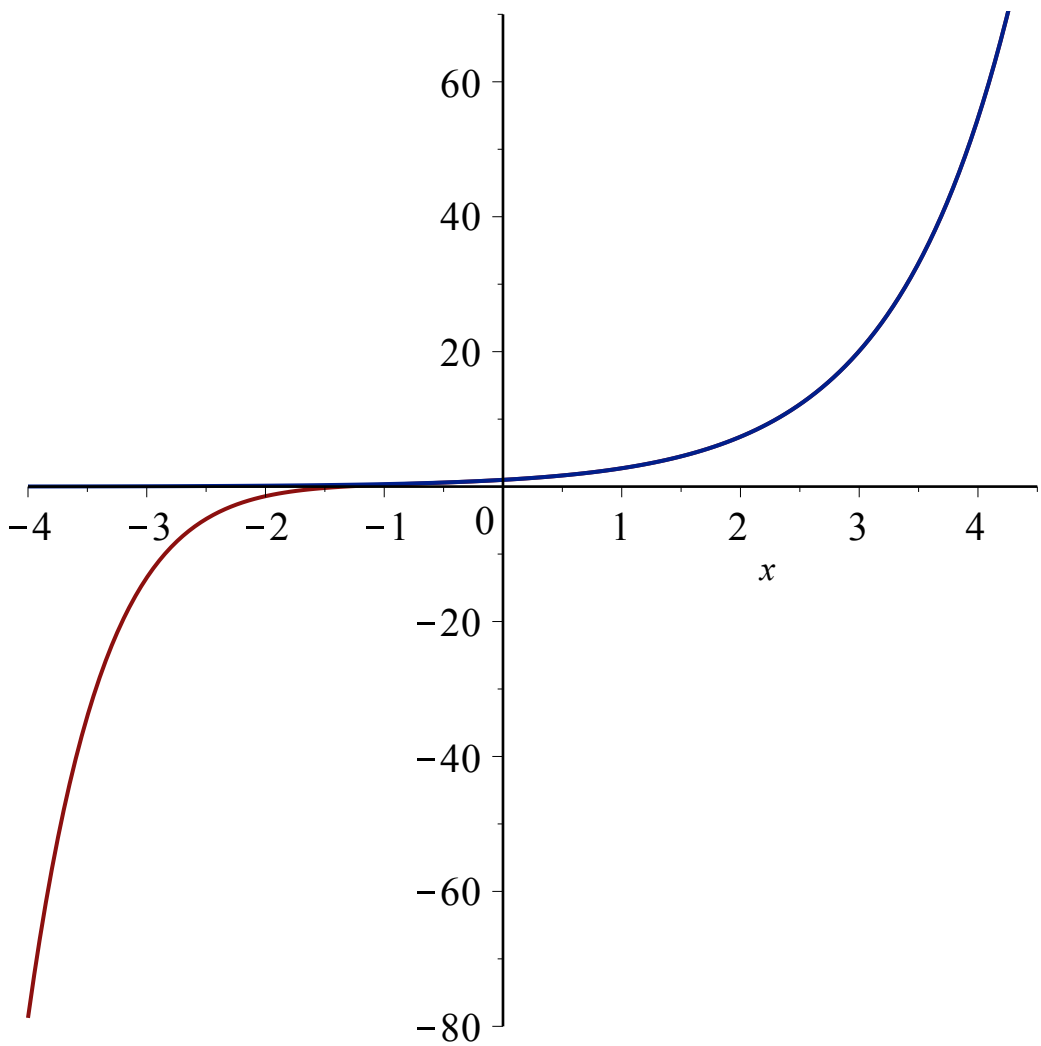
> *restart*;

taylor(exp(*x*), *x* = 2, 10);

$$e^2 + e^2 (x-2) + \frac{1}{2} e^2 (x-2)^2 + \frac{1}{6} e^2 (x-2)^3 + \frac{1}{24} e^2 (x-2)^4 + \frac{1}{120} e^2 (x-2)^5 + \frac{1}{720} e^2 (x-2)^6 + \frac{1}{5040} e^2 (x-2)^7 + \frac{1}{40320} e^2 (x-2)^8 + \frac{1}{362880} e^2 (x-2)^9 + O\left((x-2)^{10}\right) \quad \textbf{(1)}$$

$$f := x \rightarrow evalf\left(e^2 + e^2 (x-2) + \frac{1}{2} e^2 (x-2)^2 + \frac{1}{6} e^2 (x-2)^3 + \frac{1}{24} e^2 (x-2)^4 + \frac{1}{120} e^2 (x-2)^5 + \frac{1}{720} e^2 (x-2)^6 + \frac{1}{5040} e^2 (x-2)^7 + \frac{1}{40320} e^2 (x-2)^8 + \frac{1}{362880} e^2 (x-2)^9\right):$$

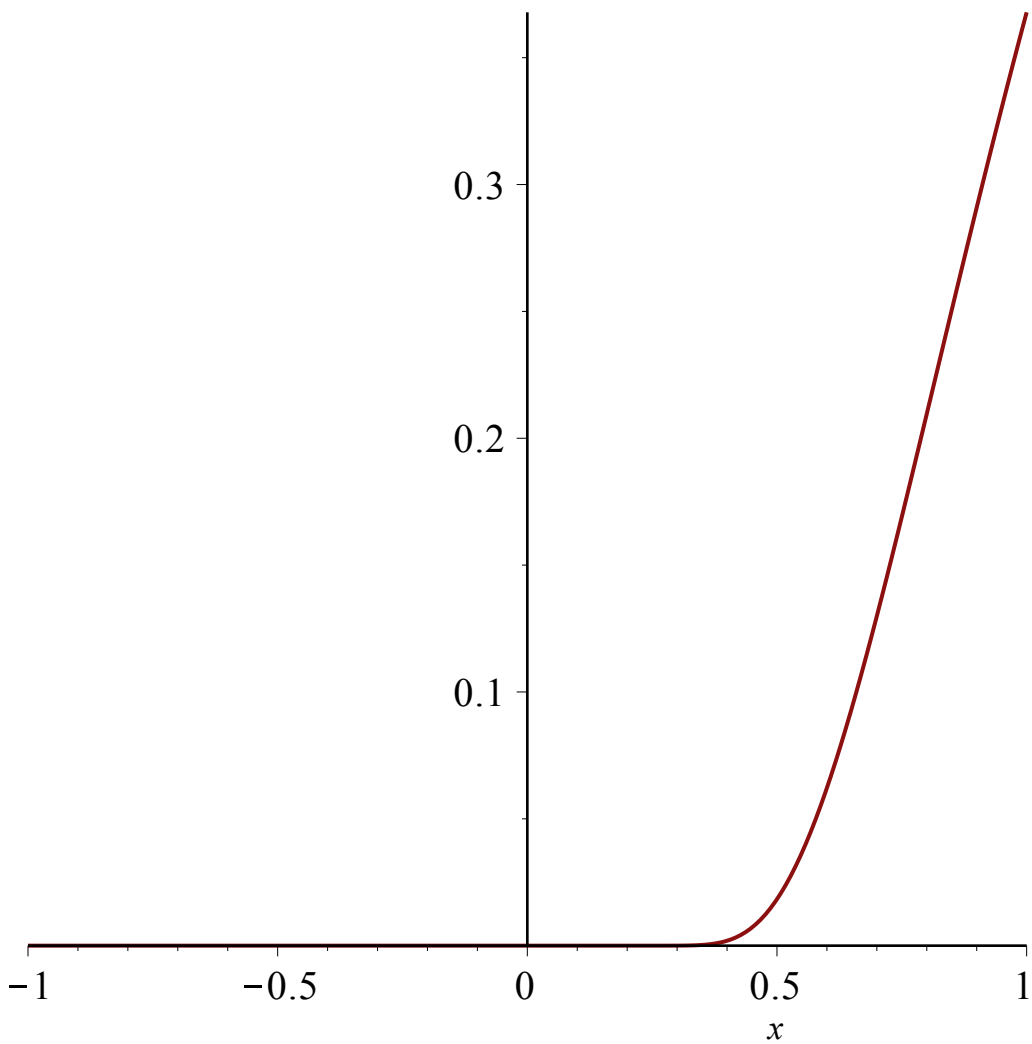
plot([*f*(*x*), exp(*x*)], *x* = −4 .. 6)



$$g := x \rightarrow \begin{cases} 0 & x \leq 0 \\ \exp\left(-\frac{1}{x^2}\right) & x > 0 \end{cases}$$

$$x \rightarrow piecewise\left(x \leq 0, 0, 0 < x, e^{-\frac{1}{x^2}}\right) \quad \textbf{(2)}$$

$plot(g(x), x = -1 .. 1)$



$taylor(g(x), x = 0, 5)$

Error, (in series/exp) unable to compute series

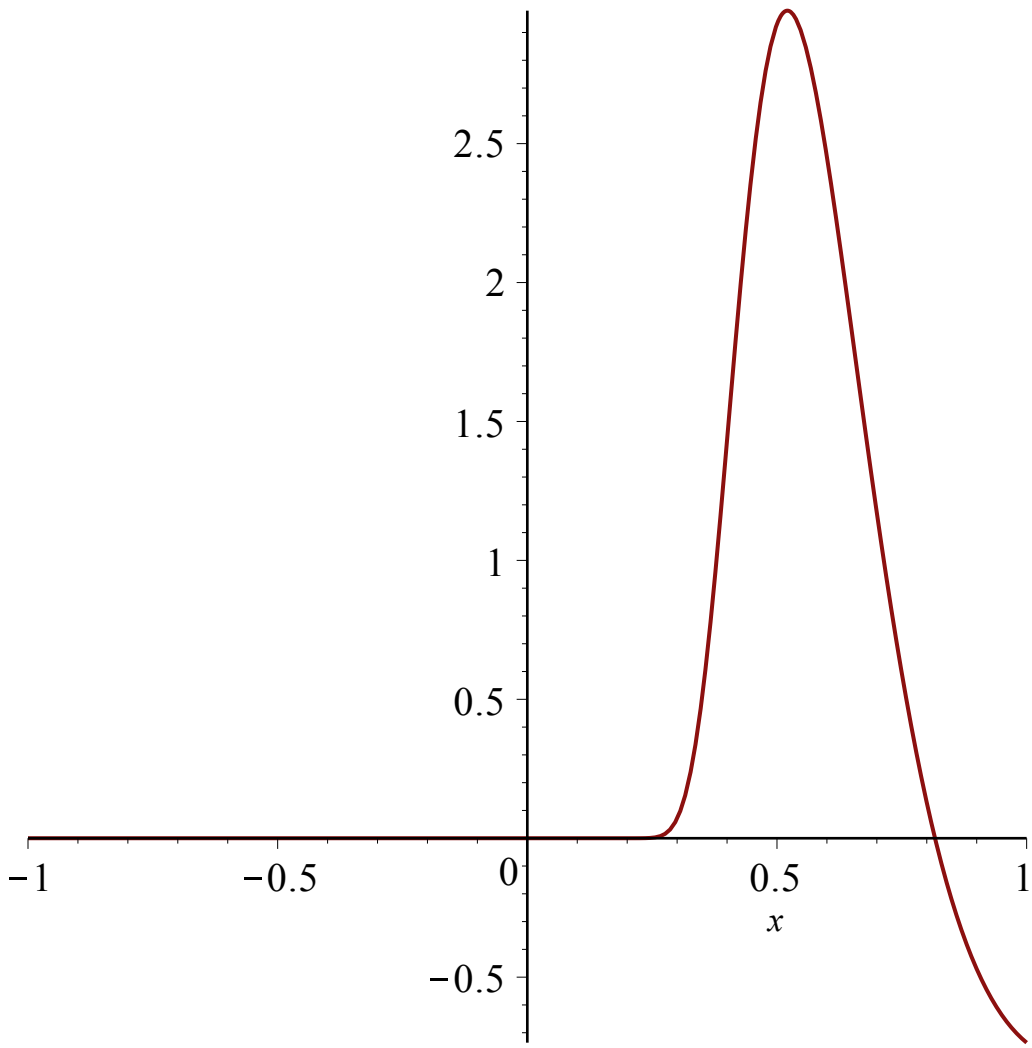
$g1 := x \rightarrow \frac{d}{dx} g(x)$

$$x \rightarrow \frac{d}{dx} g(x) \quad \textbf{(3)}$$

$g2 := x \rightarrow \frac{d}{dx} g1(x)$

$$x \rightarrow \frac{d}{dx} g1(x) \quad \textbf{(4)}$$

$plot(g2(x), x = -1 .. 1)$



$g3 := x \to \frac{d}{dx}\, g2(x)$

$$x \to \frac{d}{dx}\, g2(x) \tag{5}$$

 # the Taylor series is 0 but the function is not

$\frac{d}{dx}\, g3(x)$

$$\begin{cases} 0 & x \le 0 \\ -\frac{120\, e^{-\frac{1}{x^2}}}{x^6} + \frac{300\, e^{-\frac{1}{x^2}}}{x^8} - \frac{144\, e^{-\frac{1}{x^2}}}{x^{10}} + \frac{16\, e^{-\frac{1}{x^2}}}{x^{12}} & 0 < x \end{cases} \tag{6}$$

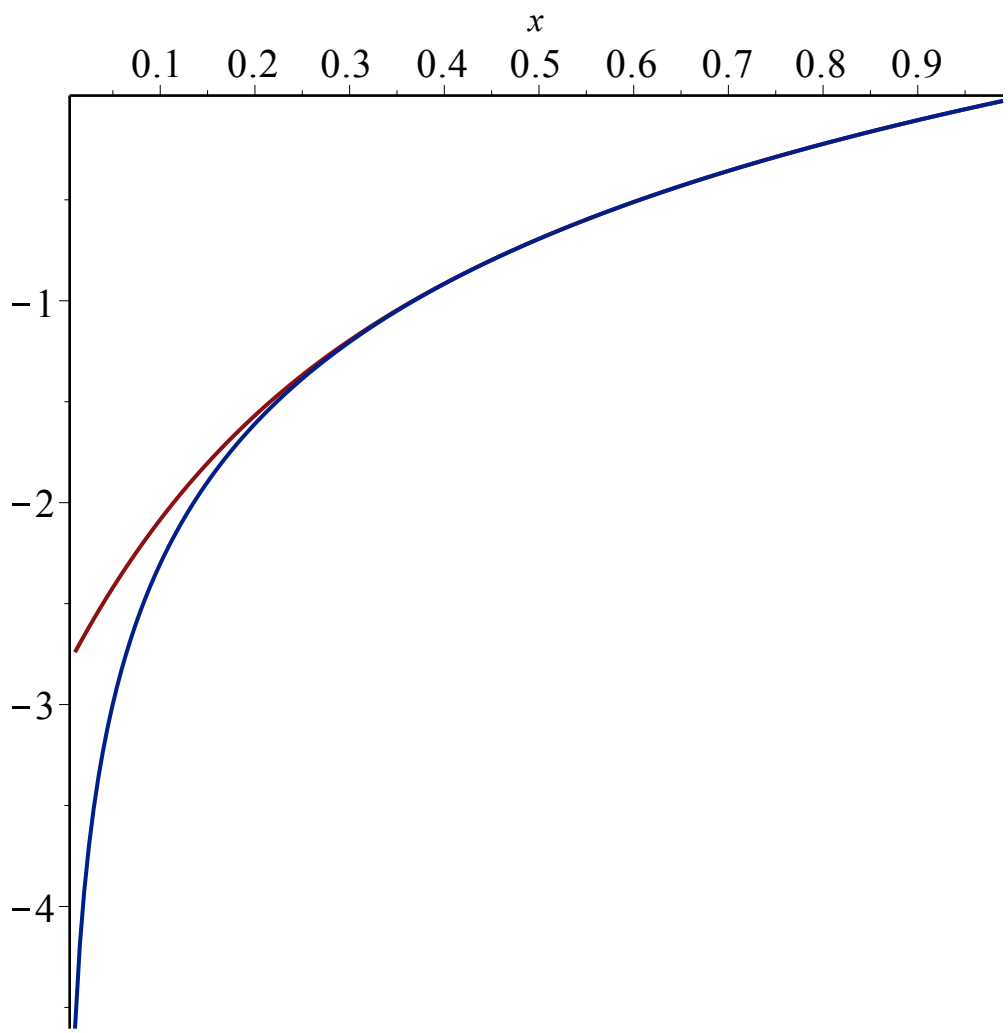
$taylor(\ln(x), x = 1, 10)$

$$x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5 - \frac{1}{6}(x-1)^6 + \frac{1}{7}(x \tag{7}$$

$$-1)^7 - \frac{1}{8}(x-1)^8 + \frac{1}{9}(x-1)^9 + O\left((x-1)^{10}\right)$$

$$h := x \to x - 1 - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5 - \frac{1}{6}(x-1)^6 + \frac{1}{7}(x-1)^7 - \frac{1}{8}(x-1)^8 + \frac{1}{9}(x-1)^9 :$$

$plot([h(x), \ln(x)], x = .01 .. .99)$



$p := x \to x^5 + 3 \cdot x^4 + 2 \cdot x^3 - 8 \cdot x^2 + 3 \cdot x + 18$

$$x \to x^5 + 3x^4 + 2x^3 - 8x^2 + 3x + 18 \quad \textbf{(8)}$$

$taylor(p(x), x = 0, 3)$

$$18 + 3x - 8x^2 + O(x^3) \quad \textbf{(9)}$$

$taylor(p(x), x = 1, 6)$

$$19 + 10(x-1) + 26(x-1)^2 + 24(x-1)^3 + 8(x-1)^4 + (x-1)^5 \quad \textbf{(10)}$$