

$$\begin{aligned} > \text{restart;} \\ > h := x \rightarrow a - x; \end{aligned} \qquad h := x \rightarrow a - x \qquad (1)$$

$$\begin{aligned} > g := x \rightarrow f(h(x)); \end{aligned} \qquad g := x \rightarrow f(h(x)) \qquad (2)$$

$$\begin{aligned} > \text{with(IntegrationTools);} \\ &[Change, CollapseNested, Combine, Expand, ExpandMultiple, Flip, GetIntegrand, GetOptions, \\ &GetParts, GetRange, GetVariable, Parts, Split, StripOptions] \end{aligned} \qquad (3)$$

$$\begin{aligned} > \\ > \\ > V := \int_0^a \frac{d}{dx} g(x) dx; \end{aligned} \qquad V := \int_0^a (-D(f)(a-x)) dx \qquad (4)$$

$$\begin{aligned} > LS := Parts(V, 1); \end{aligned} \qquad LS := f(0) - f(a) \qquad (5)$$

$$\begin{aligned} > \\ > V1 := Parts\left(V, \frac{d}{dx} g(x)\right); \end{aligned} \qquad V1 := -a D(f)(0) - \left(\int_0^a x D^{(2)}(f)(a-x) dx\right) \qquad (6)$$

$$\begin{aligned} > \\ > V2 := Parts(V1, \text{diff}(g(x), x\$2)); \end{aligned} \qquad V2 := -a D(f)(0) - \frac{1}{2} a^2 D^{(2)}(f)(0) + \int_0^a \left(-\frac{1}{2} x^2 D^{(3)}(f)(a-x)\right) dx \qquad (7)$$

$$\begin{aligned} > V3 := Parts(V2, \text{diff}(g(x), x\$3)); \\ V3 := -a D(f)(0) - \frac{1}{2} a^2 D^{(2)}(f)(0) - \frac{1}{6} a^3 D^{(3)}(f)(0) - \left(\int_0^a \frac{1}{6} x^3 D^{(4)}(f)(a-x) \right. \end{aligned} \qquad (8)$$

$$\begin{aligned} > V4 := Parts(V3, \text{diff}(g(x), x\$4)); \\ V4 := -a D(f)(0) - \frac{1}{2} a^2 D^{(2)}(f)(0) - \frac{1}{6} a^3 D^{(3)}(f)(0) - \frac{1}{24} a^4 D^{(4)}(f)(0) + \int_0^a \left(\right. \end{aligned} \qquad (9)$$

$$\begin{aligned} > V5 := Parts(V4, \text{diff}(g(x), x\$5)); \end{aligned}$$

$$\begin{aligned}
 V5 := & -a D(f)(0) - \frac{1}{2} a^2 D^{(2)}(f)(0) - \frac{1}{6} a^3 D^{(3)}(f)(0) - \frac{1}{24} a^4 D^{(4)}(f)(0) \\
 & - \frac{1}{120} a^5 D^{(5)}(f)(0) - \left(\int_0^a \frac{1}{120} x^5 D^{(6)}(f)(a-x) dx \right)
 \end{aligned} \tag{10}$$

> simplify((10));

$$\begin{aligned}
 -a D(f)(0) - \frac{1}{2} a^2 D^{(2)}(f)(0) - \frac{1}{6} a^3 D^{(3)}(f)(0) - \frac{1}{24} a^4 D^{(4)}(f)(0) \\
 - \frac{1}{120} a^5 D^{(5)}(f)(0) - \frac{1}{120} \int_0^a x^5 D^{(6)}(f)(a-x) dx
 \end{aligned} \tag{11}$$

>

> LS = (11);

$$\begin{aligned}
 f(0) - f(a) = & -a D(f)(0) - \frac{1}{2} a^2 D^{(2)}(f)(0) - \frac{1}{6} a^3 D^{(3)}(f)(0) \\
 & - \frac{1}{24} a^4 D^{(4)}(f)(0) - \frac{1}{120} a^5 D^{(5)}(f)(0) - \frac{1}{120} \int_0^a x^5 D^{(6)}(f)(a-x) dx
 \end{aligned} \tag{12}$$

> solve((12), f(a)); # note below is f(a) we have just moved equation around

$$\begin{aligned}
 a D(f)(0) + \frac{1}{2} a^2 D^{(2)}(f)(0) + \frac{1}{6} a^3 D^{(3)}(f)(0) + \frac{1}{24} a^4 D^{(4)}(f)(0) \\
 + \frac{1}{120} a^5 D^{(5)}(f)(0) + \frac{1}{120} \int_0^a x^5 D^{(6)}(f)(a-x) dx + f(0)
 \end{aligned} \tag{13}$$

>

> #f := cos(x);

$$f := x \rightarrow \begin{cases} 0 & x \leq 0 \\ \exp\left(-\frac{1}{x^2}\right) & x > 0 \end{cases};$$

$$f := x \rightarrow \text{piecewise}\left(x \leq 0, 0, 0 < x, e^{-\frac{1}{x^2}}\right) \tag{14}$$

$$\begin{aligned}
 > \text{truncated} := & a D(f)(0) + \frac{1}{2} a^2 D^{(2)}(f)(0) + \frac{1}{6} a^3 D^{(3)}(f)(0) + \frac{1}{24} a^4 D^{(4)}(f)(0) \\
 & + \frac{1}{120} a^5 D^{(5)}(f)(0);
 \end{aligned}$$

$$\text{truncated} := 0 \tag{15}$$

> plot(f(x), x = 0 .. 1);

