$f := t \to t^2$

$$f := t \mapsto t^2 \tag{1}$$

$f(3)$

$$9 \tag{2}$$

$with(Student[Calculus1])$

$$[\,AntiderivativePlot, AntiderivativeTutor, ApproximateInt, ApproximateIntTutor, ArcLength, \tag{3}$$
$\quad ArcLengthTutor, Asymptotes, Clear, CriticalPoints, CurveAnalysisTutor, DerivativePlot,$
$\quad DerivativeTutor, DiffTutor, ExtremePoints, FunctionAverage, FunctionAverageTutor,$
$\quad FunctionChart, FunctionPlot, GetMessage, GetNumProblems, GetProblem, Hint,$
$\quad InflectionPoints, IntTutor, Integrand, InversePlot, InverseTutor, LimitTutor,$
$\quad MeanValueTheorem, MeanValueTheoremTutor, NewtonQuotient, NewtonsMethod,$
$\quad NewtonsMethodTutor, PointInterpolation, RiemannSum, RollesTheorem, Roots, Rule, Show,$
$\quad ShowIncomplete, ShowSolution, ShowSteps, Summand, SurfaceOfRevolution,$
$\quad SurfaceOfRevolutionTutor, Tangent, TangentSecantTutor, TangentTutor,$
$\quad TaylorApproximation, TaylorApproximationTutor, Understand, Undo, VolumeOfRevolution,$
$\quad VolumeOfRevolutionTutor, WhatProblem\,]$

> $with(Statistics)$

$$[\,AbsoluteDeviation, AgglomeratedPlot, AreaChart, AutoCorrelation, BarChart, Biplot, \tag{4}$$
$\quad Bootstrap, BoxPlot, BubblePlot, CDF, CGF, CentralMoment, CharacteristicFunction,$
$\quad ChiSquareGoodnessOfFitTest, ChiSquareIndependenceTest, ChiSquareSuitableModelTest,$
$\quad ColumnGraph, Correlation, CorrelationMatrix, Count, CountMissing, Covariance,$
$\quad CovarianceMatrix, CrossCorrelation, Cumulant, CumulantGeneratingFunction,$
$\quad CumulativeDistributionFunction, CumulativeProduct, CumulativeSum,$
$\quad CumulativeSumChart, DataSummary, Decile, DensityPlot, DiscreteValueMap, Distribution,$
$\quad ErrorPlot, EvaluateToFloat, Excise, ExpectedValue, ExponentialFit, ExponentialSmoothing,$
$\quad FailureRate, FisherInformation, Fit, FivePointSummary, FrequencyPlot, FrequencyTable,$
$\quad GeometricMean, GridPlot, HarmonicMean, HazardRate, HeatMap, Histogram,$
$\quad HodgesLehmann, Information, InteractiveDataAnalysis, InterquartileRange,$
$\quad InverseSurvivalFunction, Join, KernelDensity, KernelDensityPlot, KernelDensitySample,$
$\quad Kurtosis, Likelihood, LikelihoodRatioStatistic, LineChart, LinearFilter, LinearFit,$
$\quad LogLikelihood, LogarithmicFit, Lowess, MGF, MLE, MakeProcedure,$
$\quad MaximumLikelihoodEstimate, Mean, MeanDeviation, Median, MedianDeviation, MillsRatio,$
$\quad Mode, Moment, MomentGeneratingFunction, MovingAverage, MovingMedian,$
$\quad MovingStatistic, NonlinearFit, NormalPlot, OneSampleChiSquareTest, OneSampleTTest,$
$\quad OneSampleZTest, OneWayANOVA, OrderByRank, OrderStatistic, PCA, PDF, Percentile,$
$\quad PieChart, PointPlot, PolynomialFit, PowerFit, PredictiveLeastSquares,$
$\quad PrincipalComponentAnalysis, Probability, ProbabilityDensityFunction, ProbabilityFunction,$

ProbabilityPlot, ProfileLikelihood, ProfileLogLikelihood, QuadraticMean, Quantile,
QuantilePlot, Quartile, RandomVariable, Range, Rank, Remove, RemoveInRange,
RemoveNonNumeric, RepeatedMedianEstimator, RousseeuwCrouxQn, RousseeuwCrouxSn,
Sample, Scale, ScatterPlot, ScatterPlot3D, Score, ScreePlot, Select, SelectInRange,
SelectNonNumeric, ShapiroWilkWTest, Shuffle, Skewness, Sort, Specialize, SplitByColumn,
StandardDeviation, StandardError, StandardizedMoment, SunflowerPlot, Support,
SurfacePlot, SurvivalFunction, SymmetryPlot, Tally, TallyInto, TreeMap, Trim,
TrimmedMean, TwoSampleFTest, TwoSamplePairedTTest, TwoSampleTTest,
TwoSampleZTest, Variance, Variation, VennDiagram, ViolinPlot, WeibullPlot,
WeightedMovingAverage, Winsorize, WinsorizedMean]

$F := x \rightarrow RiemannSum(f(t), t = 0..x, method = upper, output = value, partition = random[.001])$

$F := x \mapsto RiemannSum\left(f(t), t = 0..x, method = upper, output = value, partition = random_{0.001}\right)$ **(5)**

$F(1)$

$$0.3337154713$$ **(6)**

$F(2)$

$$2.668197729$$ **(7)**

$F(3)$

$$9.003431229$$ **(8)**

> $X := seq(1..5, .5)$

$$X := 1, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0$$ **(9)**
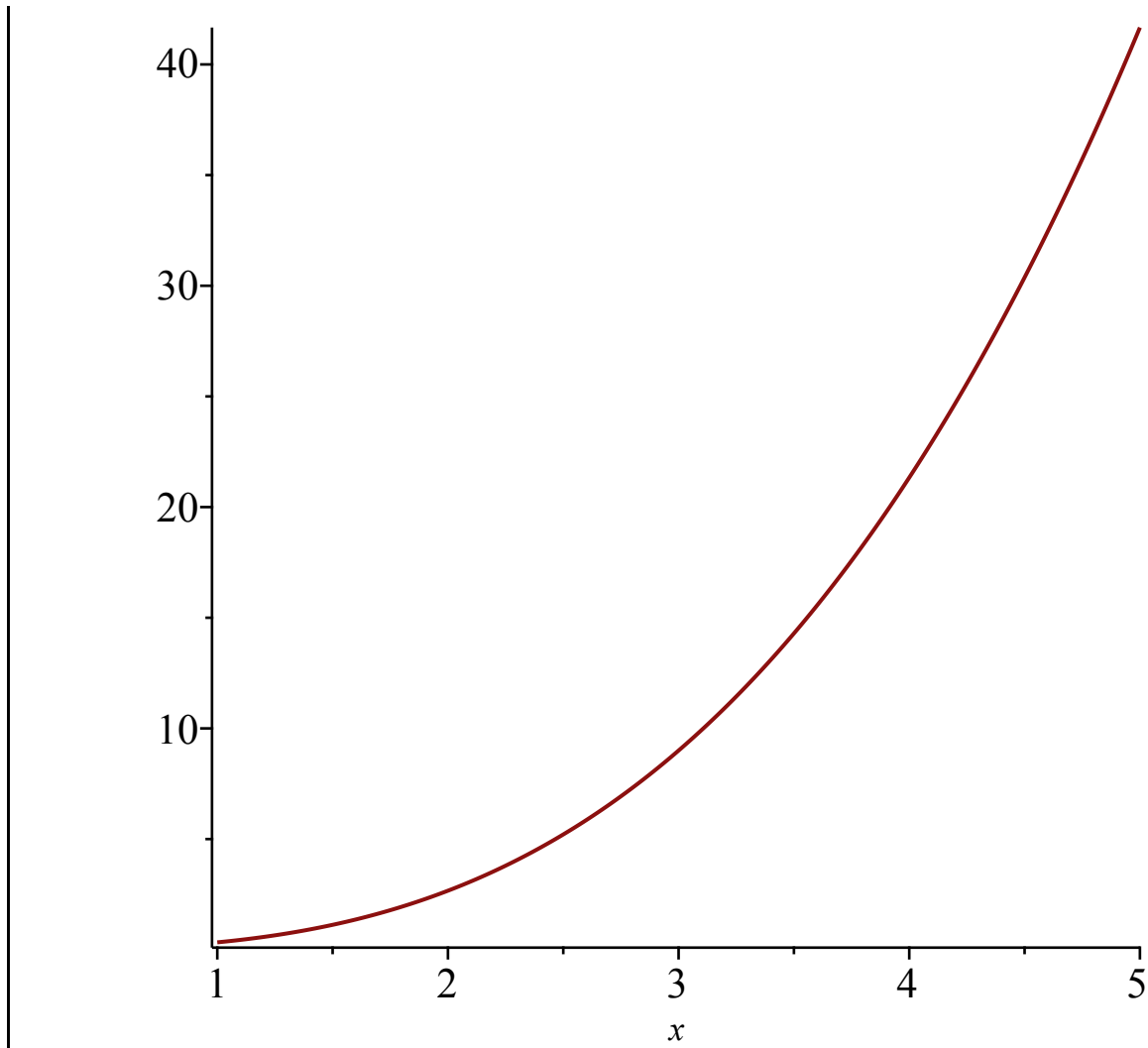
> $Y1 := seq(F(i), i = 1..5, .5)$

$Y1 := 0.3337158062, 1.125867989, 2.668190290, 5.210720816, 9.003441613, 14.29631988,$ **(10)**
$21.33944755, 30.38271705, 41.67616468$

> $ScatterPlot([X], [Y1])$

> $plot\left(\dfrac{x^3}{3}, x = 1..5\right)$

Do you expect to get x^3/3 if you start the Riemann Sum at 1 for the lower limit.
What do you expect.

>
$dx := .01$

$$dx := 0.01 \tag{11}$$

$DF2 := \dfrac{(F(2 + dx) - F(2))}{dx}$

$$DF2 := 4.023274300 \tag{12}$$

> $DF := a \rightarrow \dfrac{(F(a + dx) - F(a))}{dx}$

$$DF := a \mapsto \frac{F(a + dx) - F(a)}{dx} \tag{13}$$

> $DF(2)$

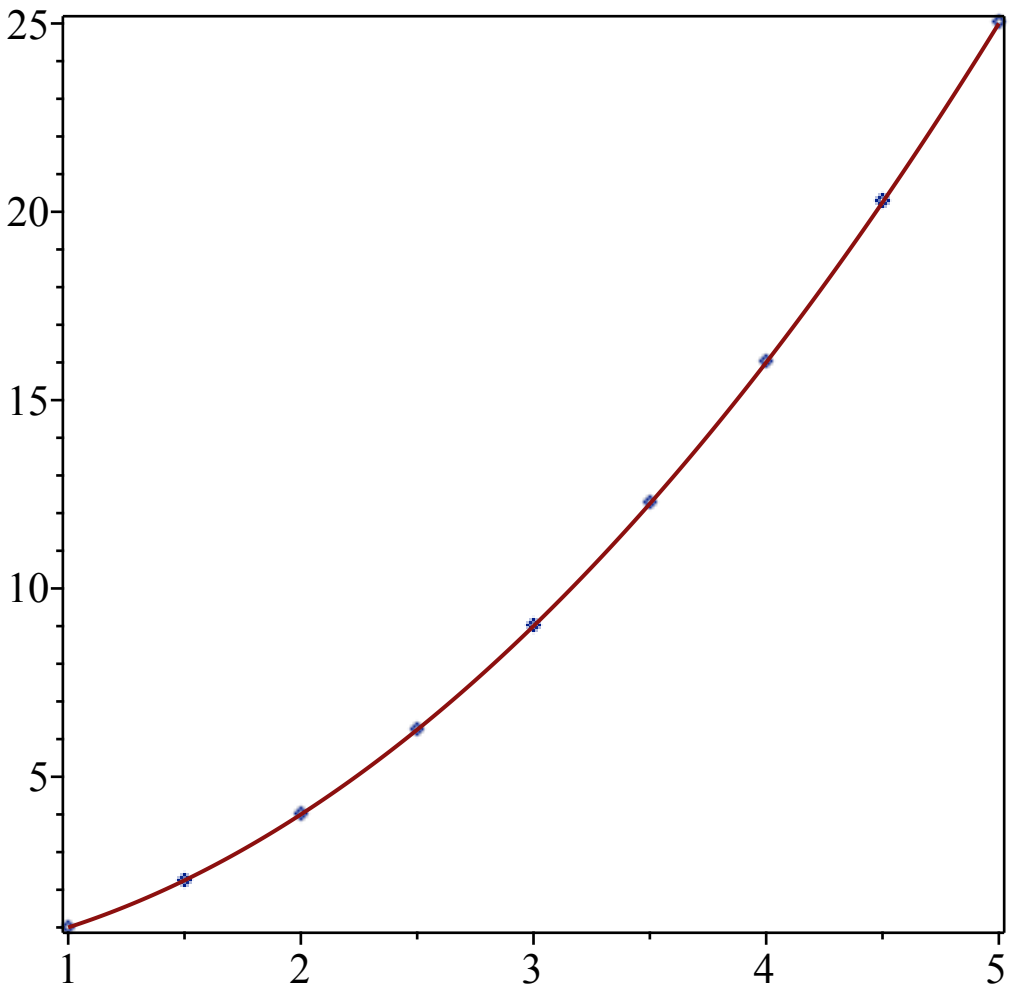$$4.020373700 \tag{14}$$

> $DF(4)$

$$16.04349000 \tag{15}$$

>

$$X := 1, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0 \tag{16}$$
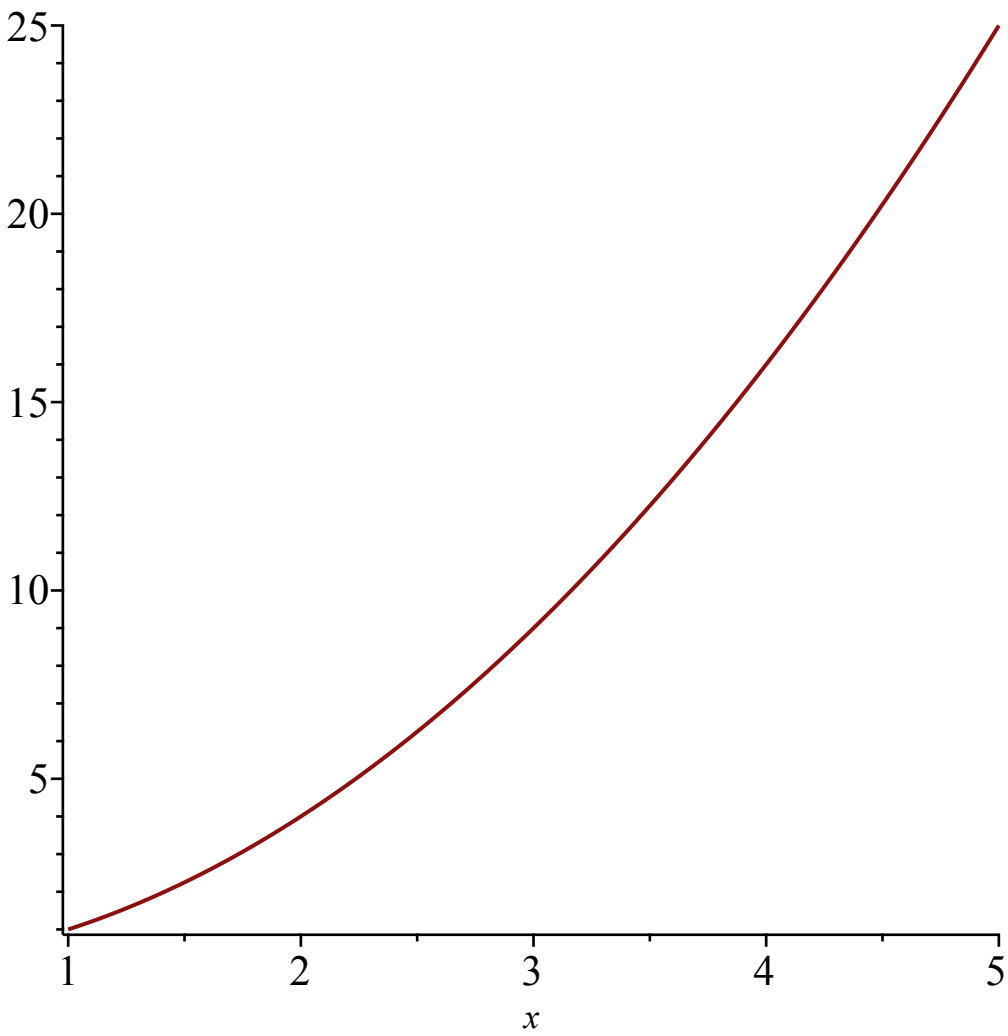
> $Y := seq(DF(i), i = 1 ..5, .5)$

$Y := 1.010721350, 2.266215300, 4.021174000, 6.276650400, 9.033147300, 12.28693000,$ **(17)**
$16.04315200, 20.30089100, 25.05119000$

>

[*AbsoluteDeviation, AgglomeratedPlot, AreaChart, AutoCorrelation, BarChart, Biplot,* **(18)**
Bootstrap, BoxPlot, BubblePlot, CDF, CGF, CentralMoment, CharacteristicFunction,
ChiSquareGoodnessOfFitTest, ChiSquareIndependenceTest, ChiSquareSuitableModelTest,
ColumnGraph, Correlation, CorrelationMatrix, Count, CountMissing, Covariance,
CovarianceMatrix, CrossCorrelation, Cumulant, CumulantGeneratingFunction,
CumulativeDistributionFunction, CumulativeProduct, CumulativeSum,
CumulativeSumChart, DataSummary, Decile, DensityPlot, DiscreteValueMap, Distribution,
ErrorPlot, EvaluateToFloat, Excise, ExpectedValue, ExponentialFit, ExponentialSmoothing,
FailureRate, FisherInformation, Fit, FivePointSummary, FrequencyPlot, FrequencyTable,
GeometricMean, GridPlot, HarmonicMean, HazardRate, HeatMap, Histogram,
HodgesLehmann, Information, InteractiveDataAnalysis, InterquartileRange,
InverseSurvivalFunction, Join, KernelDensity, KernelDensityPlot, KernelDensitySample,
Kurtosis, Likelihood, LikelihoodRatioStatistic, LineChart, LinearFilter, LinearFit,
LogLikelihood, LogarithmicFit, Lowess, MGF, MLE, MakeProcedure,
MaximumLikelihoodEstimate, Mean, MeanDeviation, Median, MedianDeviation, MillsRatio,
Mode, Moment, MomentGeneratingFunction, MovingAverage, MovingMedian,
MovingStatistic, NonlinearFit, NormalPlot, OneSampleChiSquareTest, OneSampleTTest,
OneSampleZTest, OneWayANOVA, OrderByRank, OrderStatistic, PCA, PDF, Percentile,
PieChart, PointPlot, PolynomialFit, PowerFit, PredictiveLeastSquares,
PrincipalComponentAnalysis, Probability, ProbabilityDensityFunction, ProbabilityFunction,
ProbabilityPlot, ProfileLikelihood, ProfileLogLikelihood, QuadraticMean, Quantile,
QuantilePlot, Quartile, RandomVariable, Range, Rank, Remove, RemoveInRange,
RemoveNonNumeric, RepeatedMedianEstimator, RousseeuwCrouxQn, RousseeuwCrouxSn,
Sample, Scale, ScatterPlot, ScatterPlot3D, Score, ScreePlot, Select, SelectInRange,
SelectNonNumeric, ShapiroWilkWTest, Shuffle, Skewness, Sort, Specialize, SplitByColumn,
StandardDeviation, StandardError, StandardizedMoment, SunflowerPlot, Support,
SurfacePlot, SurvivalFunction, SymmetryPlot, Tally, TallyInto, TreeMap, Trim,
TrimmedMean, TwoSampleFTest, TwoSamplePairedTTest, TwoSampleTTest,
TwoSampleZTest, Variance, Variation, VennDiagram, ViolinPlot, WeibullPlot,
WeightedMovingAverage, Winsorize, WinsorizedMean]

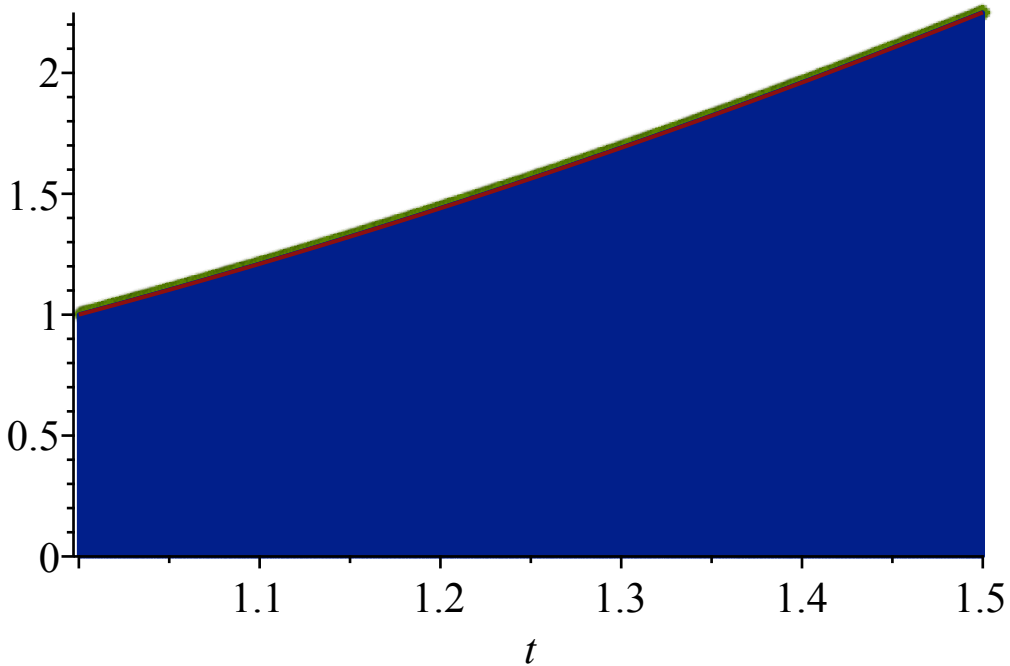**>** *ScatterPlot*([*X*], [*Y*])

> $plot(f(x), x = 1..5)$

> *FPlot* := (a, b) → *RiemannSum* (f (t), t = a ..b, method = upper, output = plot, partition = random[.001])

FPlot := (a, b) ↦ *RiemannSum* (f (t), t = a ..b, method = upper, output = plot, partition = random$_{0.001}$) **(19)**
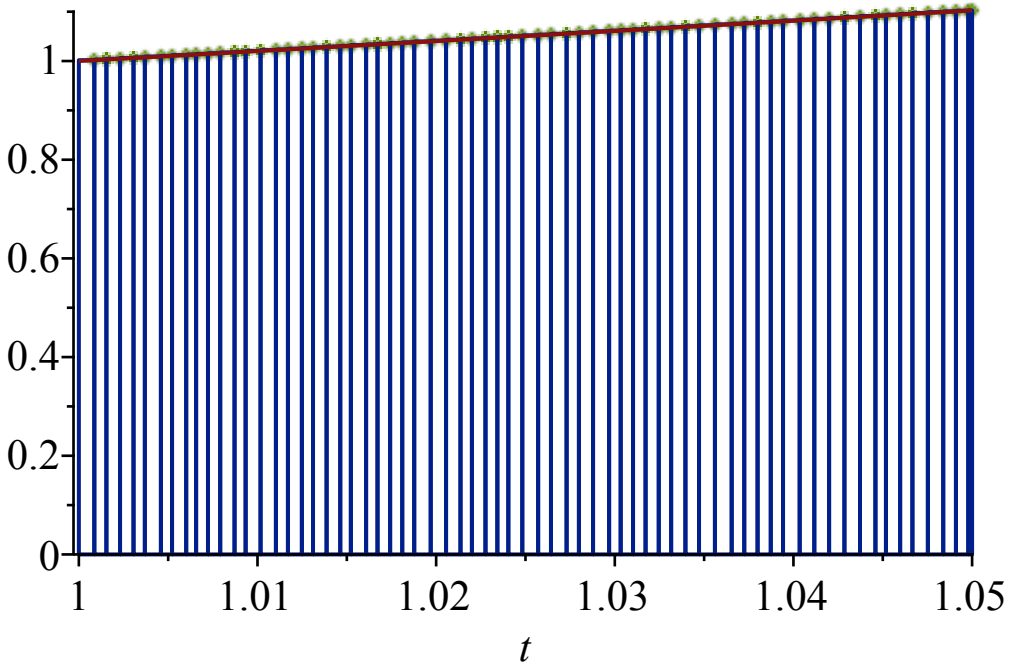
> *FPlot*(1, 1.5)

An upper Riemann sum approximation of $\displaystyle\int_{1}^{1.5} f(t)\, dt$, where

$f(t) = t^2$ and the partition is random. The approximate value of the integral is 0.7921362334. Number of subintervals used: 678.

> *FPlot*(1, 1.05)

An upper Riemann sum approximation of $\displaystyle\int_{1}^{1.05} f(t)\, dt,$

where $f(t) = t^2$ and the partition is random. The approximate value of the integral is 0.05258086246. Number of subintervals used: 67.