Taylor Series

Find the first 5 terms of the Taylor Series of given functions about the given points. Graph the functions and their Taylor series. At what points are the Taylor series bad approximations. Can you make better approximations? Take the derivatives of the Taylor polynomial and plot them against the derivatives of the functions. How well do they approximate the functions. Try with the integral of the polynomials.

$\cos(x)$; *#expanded around x=0;*

$\sin(x)$; *#expanded around x=0*

$$\sin(x) \tag{1}$$

$\exp(x)$;

 # expanded around x=0; Do you see a relationship between sin(x),cos(x) and exp(x)

$$e^{x} \tag{2}$$

$\ln(1 + x)$; *#expanded around x=0;*

$$\ln(1 + x) \tag{3}$$

$\ln(x)$; *# expanded around x=1*

$$\ln(x) \tag{4}$$

$5 \cdot x^4 + 3 \cdot x^3 + 2 \cdot x^2 + x + 6$ *#;around x=0; around x=1;*

$$5\,x^4 + 3\,x^3 + 2\,x^2 + x + 6 \tag{5}$$

$$f := x \to \begin{cases} 0 & x \le 0 \\ \exp\left(-\frac{1}{x^2}\right) & x > 0 \end{cases} \text{;\#around } x=0$$

$$x \to \mathit{piecewise}\left(x \le 0,\ 0,\ 0 < x,\ e^{-\frac{1}{x^2}}\right) \tag{6}$$

$\exp\left(x^2\right)$; *# about x=0;*

$$e^{x^2} \tag{7}$$

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cos(2*x) ; #about x=0;
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$$\cos(2\,x) \tag{8}$$

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ln(1 + cos(x));# about x=0
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$$\ln(1 + \cos(x)) \tag{9}$$

In the above questions what can you say about the remainder term at x=1;