Math 434/734

Quiz 1

Evaluate **one** of the integrals.

$$\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy$$

$$\int\int_R (x^2+y^2)^{1/2}\, dA, \quad R = \left\{(x,y), x^2+y^2 \le 9\right\}.$$

We switch the order of integration for the first integral, since we cannot integrate the function e^{x^2} in the x-variable. The region is given as horizontally simple, but we will compute the integral with the region being vertically simple. The region of integration is a triangle.

$$\int_0^2 \int_{y/2}^1 e^{x^2}\, dx\, dy = \int_0^1 \int_0^{2x} e^{x^2}\, dy\, dx = \int_0^1 [e^{x^2} y]_{y=0}^{y=2x}\, dx$$

$$= \int_0^1 e^{x^2} 2x\, dx = \int_0^1 e^u du = [e^u]_0^1 = e^1 - e^0 = e - 1.$$

with the u-substitution $u = x^2$, $du = 2xdx$.

For the second integral we use polar coordinates. The region of integration is a disc of radius 3, centered at the origin. We notice that $r = (x^2+y^2)^{1/2}$.

$$\int\int_R (x^2+y^2)^{1/2}\, dA = \int_0^{2\pi} \int_0^3 r \cdot r\, dr\, d\theta = \int_0^{2\pi} \int_0^3 r^2\, dr\, d\theta = \int_0^{2\pi} \left[\frac{r^3}{3}\right]_0^3 d\theta$$

$$= \int_0^{2\pi} \frac{3^3}{3} d\theta = \int_0^{2\pi} 9\, d\theta = [9\theta]_0^{2\pi} = 9 \cdot 2\pi = 18\pi.$$

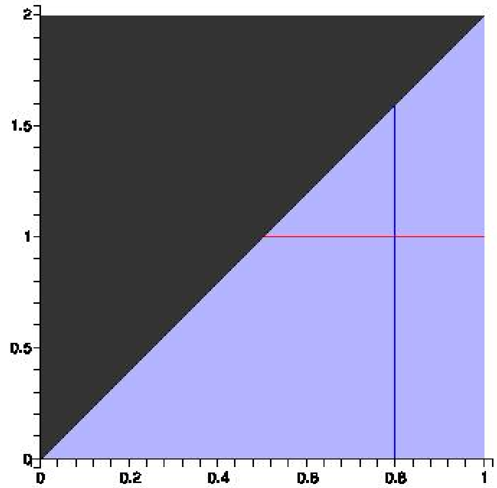

Figure 1: Region for the triangle is lightly-shaded.

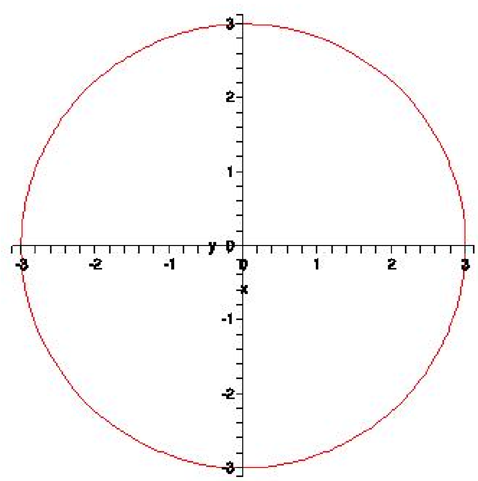

Figure 2: Region is the disc of radius 3