Math 434/734

Midterm 2

April 18, 2007

1. Use Green's theorem to evaluate

$$\int_C y dx - x^2 dy$$

where C is the square with vertices at $(0,0)$, $(2,0)$, $(2,2)$ and $(0,2)$ oriented counter-clockwise.

2. (a) Calculate the divergence of the following vector field:

$$\mathbf{F} = x^2\mathbf{i} + 2y\mathbf{j} + z\mathbf{k}$$

(b) Calculate $\int_S \mathbf{F} \cdot \mathbf{n} \, dS$, where S is the surface of the cube $[0,2] \times [0,2] \times [0,2]$ with $\mathbf{n}$ the outer unit normal vector on it.

3. (a) Calculate the curl of the following vector field:

$$\mathbf{F} = z^2\mathbf{i} + 2x\mathbf{j} - y^2\mathbf{k}$$

(b) Calculate the (line integral)

$$\int_C \mathbf{F} \cdot \mathbf{T} \, ds$$

where C is the ellipse on the xy-plane with equation $x^2 + 4y^2 = 4$ oriented counter-clockwise.

(c) (*Extra credit*) Reformulate (a) and (b) in terms of forms and redo the calculations.

4. Assume that the mixed partial derivatives of the given function are equal. Show that div (curl $\mathbf{F}$) $= 0$, which in alternative notation is $\nabla \cdot (\nabla \times \mathbf{F}) = 0$.

5. Consider the differential form $\omega = \cos y \, dx + (3y^2 - x \sin y) \, dy$. Is it closed? Is it exact? Why?