Math 434/734

Midterm 1

March 14, 2007

1. Evaluate the 1-form $dx + 2dy + 3dz$ on the oriented line segment PQ where $P(3,4,5)$ and $Q(6,-1,7)$.

2. Find the pull-back of the 1-form $\dfrac{xdx + ydy}{x^2 + y^2}$ under the map $x = r\cos\theta$ and $y = r\sin\theta$ (i.e. express the form in polar coordinates). Simplify as much as you can.

3. Evaluate **one** of the integrals.

$$\int_0^\pi \int_y^\pi \frac{\sin x}{x}\, dx\, dy, \qquad \int\int_R e^{-(x^2+y^2)}\, dA, \qquad R = \left\{(x,y), x^2 + y^2 \leq 4\right\}.$$

4. Calculate the integral of the 1-form $y^2dx + x^2dy$ along the square with vertices $(0,0)$, $(3,0)$, $(3,3)$, $(0,3)$ oriented counterclockwise.

5. Do either:

   (a) Find the 2-form describing flow in the three dimensional space from a source at $(0,0,0)$, assuming that the flow is outward at all points with a magnitude depending only on r (radial flow) and assuming there are no sources between spheres around the origin.

   Hint: The surface area of the sphere of radius r is $4\pi r^2$.

   (b) Evaluate the integral $\int_S \mathbf{F}\cdot\mathbf{n}\, dS$ (surface integral or flow) of the vector field

$$\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k},$$

   where S is the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ oriented counterclockwise as seen from the outside.