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MAT 670 **Foundations of Mathematics**

This course is part of the Mathematics Teacher Transformation Institute, an NSF Math & Science Partnership program.

COURSE DESCRIPTION

What is mathematics? What are the objects of mathematics? How are mathematical statements proved, or disproved? This course will examine fundamental material and techniques in mathematics, with an emphasis on solving equations in number systems and related notions. Traditional foundational material, from set theory to number system constructions, will be developed while simultaneously charting a fast path from the quadratic formula towards Galois Theory.

Topics to be covered include: sets and operations on sets, infinite sets and cardinality, logic and axiomatic systems, the fundamental theorem of arithmetic, examination of proof techniques, construction of the real number system, other algebraic structures, the fundamental theorem of algebra, polynomial rings, extension fields, the fundamental theorem of Galois Theory.
Weekly meetings will consist of one hour lecture and two hours of guided student exploration. Students will work collaboratively on weekly problem sets, guided by material presented in interactive lectures.

Texts: **Basic Concepts of Mathematics** *by Elias Zakon,* The Trillia group
Field Theory and Its Classical Problems *by Charles Robert Hadlock,* Carus Mathematical Monographs; No. 19.

Instructor: Rob Schneiderman
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Office hours: Mondays 12:45-2:00pm, and Wednesdays 2:00-3:45pm, Gillet Hall room 137H.

Class meetings: Wednesdays 4-7pm Carmen Hall room 325

Enrolled students can access course documents via Blackboard at: http://lca.lehman.cuny.edu/lehman/itr/html/blackboard.asp

Syllabus:
Project 1: Intro to mathematical structures
The mathematical world, sets, operations on sets, the natural numbers, prime numbers.

Project 2: Intro to number systems

More on natural numbers, operations in number systems.

Project 3: Rational and Irrational Numbers
Exponents, logs, irrational square roots.

Project 4: Mathematical Induction
Principle of induction, proofs by induction, the Fundamental Theorem of Arithmetic.

Project 5: Introduction to fields
Fields of real numbers, quadratic extensions.

Project 6: Constructible numbers
Constructing square roots, examples of constructible numbers, the plane of a field, trying to double the cube.

Project 7: Polynomials
Polynomials over a field, the Fundamental Theorem of Algebra, the division algorithm for polynomials, irreducible polynomials, greatest common divisors.

Project 8: Dimension
Vector spaces, spanning and linear independence, bases, quadratic extensions.

Project 9: Extension fields and minimal polynomials
The degree of an extension, minimal polynomials, finite extensions, extensions of extensions.

Project 10: Solutions by radicals and field automorphisms
Complex numbers, solutions in radicals, splitting fields and radical extensions,symmetries, field automorphisms, composing automorphisms.

Project 11: Groups of automorphisms
Group structure, subgroups, cyclic groups, Lagrange's theorem, Galois groups.

Project 12: Galois' Theorem
Normal subgroups, solvable groups, Galois' Theorem, Abel's Theorem.