Motivation	Offline problem	Online problem	Conclusion

Demand Smoothing Through Resource Buffering

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Motivation ●0000	Offline problem	Online problem	Conclusion
High energy de	mand		

• Key challenge for providers in electricity markets:

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Motivation ●0000	Offline problem	Online problem	Conclusion
High energy de	mand		

- Key challenge for providers in electricity markets:
 - High simultaneous demand
 - Limited supply (per unit time)

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Motivation • 0000	Offline problem	Online problem	Conclusion
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High energy demand

- Key challenge for providers in electricity markets:
 - High simultaneous demand
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- ConEd wants to sell you lots of energy...

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Motivation ●0000	Offline problem	Online problem	Conclusion
High energy de	mand		

- Key challenge for providers in electricity markets:
 - High simultaneous demand
 - Limited supply (per unit time)
- ConEd wants to sell you lots of energy... but not all right now

Motivation	
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Online problem

High energy demand

- Key challenge for providers in electricity markets:
 - High simultaneous demand
 - Limited supply (per unit time)
- ConEd wants to sell you lots of energy... but not all right now
- Extreme simultaneous usage is a challenge for the provider
- Difficult to prepare for, puts strain on grid, causes blackouts...

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Motivation ○●○○○		Offline problem	Online problem	Conclusion
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Demand charges

- One response by utilities: disincentivize peak usage
 - *peak* for the individual client
 - (other models: client incentives based on total current usage)
 - though this could be reposed from the provider's pt of view

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- Some large clients' energy bills are based on both:
 - How much kWh electricity usage charges
 - How fast (at peak) kWh/h = kW power peak charges

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Demand charges

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- Some large clients' energy bills are based on both:
 - How much kWh electricity usage charges
 How fast (at peak) kWh/h = kW power peak charges
- Per-kW peak charge \approx 100x per-kWh usage charge
 - Incentive: spread out usage over time
 - But not always possible stores have customer surges, etc.

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Alternative energy sources

• Energy sources such as solar, wind, etc., may be low-cost and clean...

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Motivation	Offline problem	Online problem	Conclusion
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Alternative energy sources

- Energy sources such as solar, wind, etc., may be low-cost and clean...
- but typically unpredictable.
- How to rely on them?

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Motivation ○○○●○ Offline problem

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Solution to both: batteries

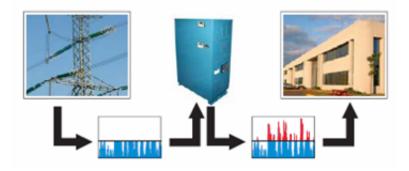


Figure: Gaia PowerTower

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Motivation 0000●	Offline problem	Online problem	Conclusion

- Resource is water
 - $\bullet \ \ \mathsf{Power \ Tower} \to \mathsf{water \ tower}$

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Motivation	Offline problem	Online problem	Conclusion
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- Resource is water
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- Resource is undistinguished unit-time jobs doable in advance

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Motivation	Offline problem	Online problem	Conclusion
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Resource is water

- Power Tower \rightarrow water tower
- Resource is *undistinguished* unit-time jobs doable in advance
- Resource is unsold products

The Xbox shortage of 2005

"[T]he Xbox 360 can be produced only gradually, but all the demand is there at once. Plentiful supply would be possible only if Microsoft made millions of consoles in advance and stored them without releasing them, or if it built vast production lines that only ran for a few weeks–both economically unwise strategies. ... The steady supply can't match peak December demand." (http://www.slate.com/id/2132071/)

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Motivation	Offline problem	Online problem	Conclusion
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• Or can it?

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Motivation	Offline problem •oooooooo	Online problem	Conclusion
Model and n	otation		

- At each time: How much extra to request? Or how much less?
- Goal: make request curve as smooth as possible (min max)
 - While always satisfying demand
 - Ideally without wasting any energy
- A dilemma:
 - Request nothing extra: waste the battery
 - Request too much extra: introduce new peaks
- Obj ftn is *max*, not sum
 - "strict liability"

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Motivation	Offline problem ○●○○○○○○○	Online problem	Conclusion

Offline problem definition

Notation

- n discrete timesteps
- d_i : demand at time *i* (demands = input)
- r_i: request at time i (requests = output)

•
$$D = \max_i d_i$$

•
$$R = \max_i r_i$$

• b_i : battery level at start of time i ($b_1 = 0$ or $b_1 = B$)

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$$R = \max_i r_i$$

• b_i : battery level at start of time i ($b_1 = 0$ or $b_1 = B$)

Goal: choose requests r_i to minimize R

- i.e., make request curve as smooth as possible
 - all demands must be satisfied
 - with no underflow: $\forall i \ b_i \geq 0$
- NB: $b_{i+1} = b_i + r_i d_i$ (except when underflow/overflow)

Motivation	
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Offline problem

Online problem

Incorporating the free source

- At each time may also have free source amount f_i
- In this case, effective demand is $\hat{d}_i = d_i f_i$
- As long as negative demands make sense, can ignore free source wlog

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Optimal solution (offline, unbounded battery)

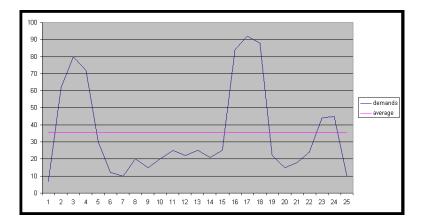


Figure: Demands and mean

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Optimal solution (offline, unbounded battery)

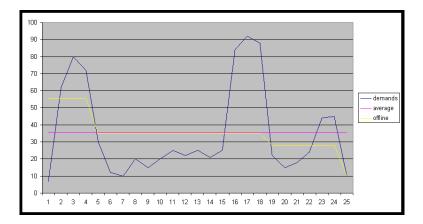


Figure: Demands, mean, and optimal

Motivation	Offline problem	Online problem	Conclusion
Threshold al	gorithms		

- All our algorithms are based on thresholds
 - $\bullet~\mbox{Threshold} = \mbox{amount the algorithm tries to request}$
 - Offline: global threshold T
 - Online: threshold T_i at timestep i
- At each time, (try to) request T_i , and charge/discharge the rest (based on $d_i \& b_i$)

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Motivation	Offline problem ○○○○○●○○○	Online problem	Conclusion
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- At each time, (try to) request T_i , and charge/discharge the rest (based on $d_i \& b_i$)

Two issues:

- Overflow: battery too full: ok, just lose the energy
 - Or just request less
- Underflow: battery below empty: forbidden ("crash")

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Offline problem

Online problem

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Threshold algorithms

for each timeslot *i*
if
$$T_i > d_i$$

charge $min(T_i - d_i, B - b_i)$
else
discharge $d_i - T_i$

Figure: Threshold algorithm schema (assumes $b_i \ge d_i - T_i$)

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Motivation	Offline problem ○○○○○○○●○	Online problem	Conclusion
Offline problem	IS		

- Two subsettings: unbounded and bounded batteries
- Both solvable by LP
 - But we seek efficient combinatorial algorithms
 - Our online algorithms will use offline as subroutine
 - Initial/final conditions: slightly preprocess input (demands)

Motivation	Offline problem ○○○○○○○●○	Online problem	Conclusion
Offline problem	S		

- Two subsettings: unbounded and bounded batteries
- Both solvable by LP
 - But we seek efficient combinatorial algorithms
 - Our online algorithms will use offline as subroutine
 - Initial/final conditions: slightly preprocess input (demands)
- Unbounded battery: find hardest prefix (average) of demands
 - For $b_1 = 0$ case
 - Easy in linear time

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Motivation	Offline problem ○○○○○○○●	Online problem	Conclusion
Offline problems	S		

• Bounded battery: find hardest subsequence (critical region)

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Offline problem	Online problem	Conclusion
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Offline problems

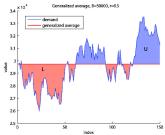
- Bounded battery: find hardest subsequence (critical region)
- In this region, for OPT:
 - battery will go from full to empty (if ever does)
 - requests will be flat
 - request value: $(-B + \sum_{t=i}^{j} d_t)/(j i + 1)$ ("generalized average" or GA)

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Offline problems

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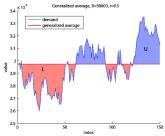


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Offline problems

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• Can easily find this region in quadratic time

Motivation	Offline problem	Online problem	Conclusi

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Motivation	Offline problem	Online problem	Conclusion
Online algori	thms		

- Change: demands *d_i* now arrive online (free source values, too)
- Goal: competitiveness with OPT

 $\mathbf{\Box}$

• Potential obj ftns: minimize peak, or maximize savings

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Motivation	Offline problem	Online problem	Conclusion
Online algorith	ms		

- Change: demands d_i now arrive online (free source values, too)
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 - Common intuition: maybe the future will be like the past

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Online algorith	ms		

- Change: demands *d_i* now arrive online (free source values, too)
- Goal: competitiveness with OPT
 - Potential obj ftns: minimize peak, or maximize savings
- One idea: alpha policy [Hunsaker et al. 1998]
 - Common intuition: maybe the future will be like the past
- $\rightarrow\,$ at each moment, run OPT on the full history up until now
 - Then choose accordingly
 - i.e., request OPT's max-so-far (times some $\alpha \geq 1$)
 - unbounded case: just the maximum prefix mean

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Motivation	Offline problem	Online problem ○●○○○○○○○	Conclusion



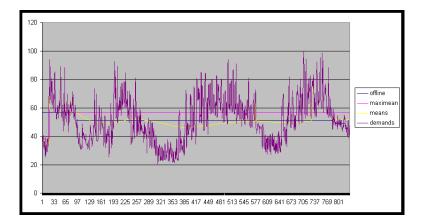


Figure: Demands, mean, and optimal

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Motivation	Offline problem	Online problem ○○●○○○○○○	Conclusion

Request graph: means

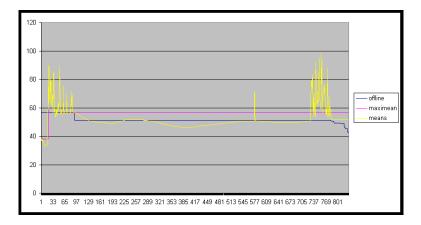


Figure: Demands, mean, and optimal

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Motivation	Offline problem	Online problem	Conclusion
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Request graph: maximean

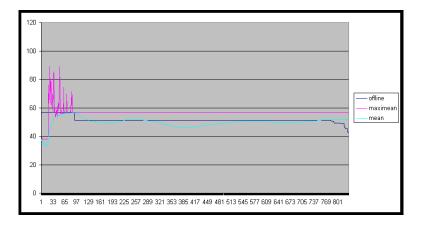


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Motivation	Offline problem	Online problem	Conclusion
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Competitive online algorithm?

Unfortunately, there are competitiveness counterexamples for both the *minimize peak* and *maximize savings* problems

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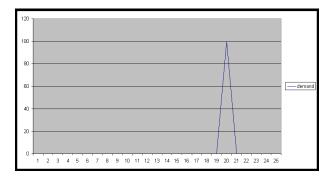


Figure: Competitiveness counterexample for $b_1 = 0$ case

Motivation	Offline problem	Online problem	Conclusion
Semi-online	algorithms		

- So, relaxation for *maximize savings* problem: assume we can guess peak demand *D* (e.g. from history data)
 - for *minimize peak* problem: still factor-*n* lower bound on competitiveness

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Motivation	Offline problem	Online problem	Conclusion
Semi-online al	gorithms		

- So, relaxation for *maximize savings* problem: assume we can guess peak demand *D* (e.g. from history data)
 - for *minimize peak* problem: still factor-*n* lower bound on competitiveness
- Now do "alpha from above" (Alg 2.a)
- Since opt savings is $D R_{opt}$
 - Always request so that savings is exactly $1/H_n$ of "optimal savings so far" (compared to D)
 - Alg 2.a: $T_i \leftarrow D \frac{D \hat{\mu}(1, i)}{H_n}$

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Semi-online alg	orithms		

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 - Alg 2.a: $T_i \leftarrow D \frac{D \hat{\mu}(1,i)}{H_n}$
- *H_n*-competitive by construction, *assuming it's correct*
 - i.e., assuming battery never crashes
 - i.e., request T_i always suffices, with no underflow

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Motivation	Offline problem	Online problem ○○○○○○●○○	Conclusion

Lemma

If there is an instance with underflow for Alg 2.a, then there will be one with battery decreasing from full to empty, with no overflow in the middle.

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Lemma

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In this case, we have: $b_{i+1} = b_i + r_i - d_i$.

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Thm: Alg 2.a is correct, and so H_n -competitive

Proof sketch: We have H_n -approx by construction, as long as no underflow. Cite lemma. But for such monotonic instances, total net discharge is $\leq B$. Thus the final battery level is nonnegative.

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NB: applies to both bounded and unbounded battery.

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Thm: H_n -competitive is optimal for bounded battery

Proof sketch: If ALG is *c*-competitive $(c \ge 1)$ and $b_1 = B$, can force it to discharge:

- *B*/*c* at time 1,
- B/(2c) at time 2,
- B/(3c) at time 3, etc.

Total discharge: $\sum_{i} B/(i \cdot c) = H_n \cdot B/c$.

The demand sequence is just: (D, D, D, ..., D), for some $D \ge B$.

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The demand sequence is just: (D, D, D, ..., D), for some $D \ge B$.

For unbounded battery, have lower bound of $H_n - 1/2$.

Motivation	Offline problem	Online problem ○○○○○○○●	Conclusion
Semi-online alg	orithms		

- NB: in some sense, n (hence H_n) is constant
 - monthly billing periods, coarseness of time units...

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- Semi-online algorithms
 - NB: in some sense, n (hence H_n) is constant
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 - Alg 2.a can be $O(n^2)$
 - Basically computing OPT over time
 - At each time, extend O(n) GAs

Motivation		Offline problem	Online problem ○○○○○○○●	Conclusion
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- Semi-online algorithms
 - NB: in some sense, n (hence H_n) is constant
 - monthly billing periods, coarseness of time units...
 - Alg 2.a can be $O(n^2)$
 - Basically computing OPT over time
 - At each time, extend O(n) GAs
 - Turns out (lemma) we can do better with less work: O(n)
 - Suffices to find the GA back to last time battery was full (for us)
 - Forget about prefix: $n \rightarrow n' < n$
 - More importantly: only one GA to extend each time
 - Alg 2.b: $T_i \leftarrow D \frac{D \mu(s_i, i)}{H_{(n-s_i+1)}}$
 - Analysis same as for Alg 2.a

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Motivation	Offline problem	Online problem	Conclusion
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Summary

- Gave poly-time offline algorithms for bounded and unbounded batteries
- Unfortunately, many online problem settings here are intractable, but not all
- Gave O(1)-per-unit-time online algorithms for two of them

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Motivation	Offline problem	Online problem	Conclusion
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Future directions

Problem extensions:

- Entry loss
 - Corresponding online algorithm also appears to be H_n -competitive (WEA '08), but no proof
- Self-discharge (batteries draining over time)
- 30-minute rolling averages

Experimental work:

• Tuning more aggressive algorithms to empirical data (WEA '08)

Other models:

• E.g. dynamic pricing based on total current demand

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Motivation	Offline problem	Online problem	Conclusion
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Thanks!

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