

Efficient Multicast in Hybrid Wireless Networks

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Abstract—The increasing popularity of smartphones and other similar multi-modal wireless devices has created an opportunity for the realization of large-scale hybrid (or heterogeneous) networks. Typically, modern mobile devices are likely to support a short range communication interface (e.g. IEEE 802.11/WiFi) and/or a longer range communication interface (e.g. cellular data link wireless technology). Multi-hop wireless networking over WiFi can help to extend the range of cellular networks in low SINR regions as well as to alleviate network congestion. Conversely, equipping a few nodes in a mobile ad hoc network (MANET) with cellular radios can help to heal wireless network partitions and, thus, to improve the overall network connectivity. One can envision large scale group communication (or multicast) applications including real-time video conferencing (e.g., iPhone FaceTime), P2P video and file sharing, and “voice call groups” in disaster relief and military hybrid networks. In this paper, the problem of resource-efficient multicast in hybrid wireless networks which include both point-to-point (cellular) and broadcast (MANET) links is considered. The underlying optimization problem is a hybrid of two well-known NP-hard graph optimization problems—the Minimum Steiner Tree problem (for point-to-point links) and the Minimum Steiner Connected Dominating Set problem (for broadcast links). We consider both edge- and node-weighted versions of this problem and use distinctly different methodologies to give three algorithms with guaranteed approximation factors. We further demonstrate by means of simulation modeling of standard deployment scenarios that while one algorithm outperforms another in terms of the tree cost, the latter outperforms the former in terms of complexity and other practical considerations.

I. INTRODUCTION

The increasing popularity of smartphones and other similar multi-modal wireless devices have created an opportunity for the realization of large-scale hybrid (or heterogeneous) networks. Typically, modern mobile devices are likely to support IEEE 802.11/WiFi, which have a short communication range, and/or cellular data link wireless technologies, which have a longer communication range.

Multi-hop wireless networking over WiFi can help to extend the range of cellular networks in low SINR (Signal to Interference plus Noise Ratio) regions as well as to alleviate network

congestion [15]. Conversely, equipping a small fraction of nodes in a mobile ad hoc network (MANET) with cellular radios can help heal wireless network partitions and, thus, improve the overall network connectivity. This proves useful in the military and sensor networks/DTN contexts where islands of connectivity are formed no matter how hard one tries to make the MANET connected, e.g., CERDEC MACE program [5]. One can also envision large scale group communication (or multicast) applications including real-time video conferencing (e.g., iPhone FaceTime), P2P video and file sharing, and “voice call groups” in disaster relief and military hybrid networks.

In this paper, we consider the problem of resource-efficient multicast in hybrid wireless networks which include both point-to-point (cellular) and broadcast (WiFi) links. The underlying optimization problem is a hybrid of two well-known NP-hard graph optimization problems—the Minimum Steiner Tree problem (for point-to-point links) and the Minimum Steiner Connected Dominating Set problem (for broadcast links). We consider both edge- and node-weighted versions of this problem and use two distinctly different methodologies to give two algorithms with guaranteed approximation factors.

Network multicast is an important topic to study in such hybrid networks. Smartphone users like sharing real-time video or want to perform videoconferencing with friends, which forms a multicast group [13]. Even without P2P technologies enabled on the smartphone, one can imagine a simple use case of a hybrid network consisting of 4G cellular (point-to-point) + 802.11 WiFi (last hop broadcast) [11]. Other examples of multicast include “voice call groups” in military and disaster relief networks, and video sharing in P2P wireless is a popular problem [9]. Finally, “link state” routing algorithms may need efficient network-wide broadcast of link state updates.

We believe that the following categories of hybrid networks stand to benefit from efficient multicasting:

Tactical Networks. Mission-critical operations for field and disaster relief missions often rely on a combination of wireless ad hoc networks (which can usually be swiftly deployed) and on-site cellular networks (these may be limited in availability). An example of such a network is described in [5].

Mobile Content Distribution Networks. The proliferation of mobile gadgets facilitates peer-to-peer communication among users, which may be more economical than, but not as reliable as the wireless data connection through base stations.

Vehicular Networks. The advance of vehicular technology en-

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ables dedicated short-range communications (DSR) equipped in the emerging new vehicles to receive traffic alerts. It is also possible to employ DSR for distributing wide-area traffic information among vehicles, in addition to connections through road-side kiosks.

While the set of various available technology types of the nodes in the hybrid network can be very general, in *wireless hybrid networks*, nodes could typically possess one or more of the following distinct types of wireless technologies:

Cellular wireless via base stations (macrocells, femtocells etc.)—in this case, any two smartphones equipped with the cellular SIM card can potentially communicate via a series of base stations (BS). The quality of this “cellular” link between two smartphones will depend on how far they are located with respect to their closest BSs, respectively. Thus, the set of cellular-capable nodes form a weighted *clique*. This can be generalized to a non-clique where broadcast technologies in cellular networks [8] can be exploited as described in Section VI; or even a disconnected graph.

Wireless point-to-point mesh—in this case, any two nodes equipped with the specific interface hardware can potentially communicate directly if they are within the transmission range of each other. However, a node does not communicate with multiple nodes in range using a single broadcast transmission. The set of mesh nodes form a *random geometric graph*.

MANET wireless broadcast—this is similar to the wireless mesh setting with the exception that a node can reach multiple nodes in its transmission range using a single broadcast transmission. The set of MANET nodes form a *random geometric directed hypergraph*.

A. Our Contributions

In this paper, we focus on hybrid networks composed of cellular and MANET technologies since they represent point-to-point wireless and wireless broadcast advantage. We explore theoretical/algorithmic aspects of the hybrid network multicast problem using standard deployment modelling scenarios. Specifically, we propose and study a directed hypergraph model for edge-weighted hybrid networks and give an approximation algorithm for a minimum cost multicast based on the approximation algorithm for the *directed* Steiner tree problem by Charikar et al. [4]. We also propose a $(3.5 \ln |\mathcal{M}| + \theta(1))$ -approximation algorithm for the unweighted hybrid network multicast problem, which tries to minimize the total number of transmissions irrespective of the wireless network costs (we derive a pseudo-polynomial approximation ratio for the heuristic algorithm when the unweighted algorithm is applied to a weighted version of the problem), and another $(2 \ln |\mathcal{M}| + \theta(1))$ -approximation algorithm for same setting. Finally, we perform a systematic evaluation of tradeoffs between the performance of the aforementioned algorithms in realistic wireless hybrid network scenarios.

B. Related Work

Hybrid wireless networks are a well researched topic [10], [14], [17], but the problem of multicast in such networks

has not received adequate attention. Our optimization problem is a hybrid of Steiner Tree problem for point-to-point links and Steiner Connected Dominating Set (StCDS) problem for broadcast links, both of which are NP-hard. Efficient algorithms for solving Steiner CDS problem, which captures the wireless version of multicast problem, was first studied by Guha and Khuller [6]. In their paper, they presented a polynomial-time $O(\log n)$ -approximation algorithm for the *unweighted* Steiner CDS problem, and showed a setting in which that there exists no polynomial-time algorithm of any polynomial approximation factor for the *weighted* Steiner CDS problem. Wu, Xu, and Chen [16] presented a slightly improved algorithm for Steiner CDS problem. In planar or unit-disk graphs, CDS problems are more tractable, approximable within a constant factor. However, these graphs cannot realistically capture hybrid networks, especially because the wireline part may not be restricted by the geometry. Panigrahi [12] presented a class of problems called activation network design problem, in which the cost of a link depends on the states of its end-nodes through an activation function. The activation network problem does not precisely capture the hybrid network multicast problem, although CDS can be mapped to an activation network problem with a constant approximation ratio.

II. PRELIMINARIES AND PROBLEM FORMULATION

The description of the traditional multicast problem and associated terminology can be found in [3], [8], [16], [6]. We now present a generalised hybrid multicast network problem. Consider a hybrid network represented by a multigraph $G = (V, E)$ characterized as follows. The vertex set V is split into three partitions: V_{manet} (for wireless MANET nodes—*type M* for short), V_{cell} (for wireless cellular nodes—*type C* for short), and V_{hyb} (for hybrid nodes with both MANET and cellular interfaces—*type H* for short). Each link $(u, v) \in E$ represents a communication link between a pair of nodes, where E is split into the following two partitions: E_{manet} (for wireless MANET links) and E_{cell} (for cellular links), where

$$\begin{aligned} (u, v) \in E_{\text{manet}} &\Rightarrow u, v \in V_{\text{manet}} \cup V_{\text{hyb}}, \\ (u, v) \in E_{\text{cell}} &\Rightarrow u, v \in V_{\text{cell}} \cup V_{\text{hyb}}. \end{aligned}$$

Denote by $H = (V(H) \subseteq V, E(H) \subseteq E)$ a subgraph of G and consider a set of terminals $\mathcal{M} \subseteq V$. Denote the set of leaf nodes in a subgraph H by $\text{Leaf}(H)$. Also, the set of nodes connected by wireless broadcast links in H is defined by:

$$\begin{aligned} V_{\text{manet}}(H) \triangleq \{v \in (V_{\text{manet}} \cup V_{\text{hyb}}) \cap V(H) \mid \\ \exists (u, v) \in E_{\text{manet}} \cap E(H)\} \end{aligned}$$

Let $c(v)$ and $\gamma(e)$ denote the costs of a node v and a link e , respectively. In this paper we consider scenarios with *additive costs*, i.e., where the cost of a multicast subgraph is the *sum* of the costs of the nodes and edges contained in that subgraph. An important example of such a family of additive costs is “energy”. Other nonadditive multicast costs such as “latency” are left for future research.

Given a subgraph H , we define the cost as a sum of costs due to all MANET broadcasts (which are captured in terms of non-leaf node costs since a MANET node has to pay the cost for all its adjacent MANET links only once) or cellular point-to-point links; since the leaf nodes do not incur any cost in transmitting a message, their costs are not counted:

$$\text{Cost}(H) \triangleq \sum_{v \in V(H): v \notin \text{Leaf}(H) \cap V_{\text{manet}}(H)} c(v) + \sum_{e \in E(H) \cap E_{\text{cell}}} \gamma(e)$$

Generalized hybrid multicast network problem consists of minimizing $\text{Cost}(H)$ with respect to H subject to the following: (i) H is a connected subgraph of G , and (ii) each pair of nodes $u, v \in \mathcal{M}$ are connected via H .

III. HYBRID NETWORK MODELS AND APPROACH

Both edge and node based costs are possible in the problem formulation presented in Section II. Owing to the presence of both broadcast and point-to-point links in a hybrid network, care must be taken to model the costs appropriately.

Typically, there are a few different types of costs that may be encountered in hybrid networks. First, when a MANET or a hybrid node v performs a wireless broadcast, all the MANET links adjacent to this node are simultaneously activated, hence the cost charged to a broadcast can be transferred from the links to the node instead. Secondly, when a cellular or a hybrid node v performs a transmission over a cellular link to another cellular or hybrid node, each cellular link incident on v constitutes a separate cost. Finally, additional costs may be charged to nodes during the multicasting operation.

We propose two somewhat related but distinct graph constructions for addressing both the cases when only links are charged a cost and when both links and nodes are charged costs.

A. Constructing Edge-weighted Graphs from Directed Hypergraphs

Consider a *directed hypergraph* $H = (V, E)$ where V is a set of *vertices* and E is a set of *hyperedges*. A hyperedge $e \in E$ is an ordered pair (V_T, V_H) of nonempty, disjoint subsets of V referred to as the *tail vertices*, $V_T(e)$, and the *head vertices*, $V_H(e)$, of the hyperedge e , respectively. For each hyperedge e we associate a cost $\gamma(e) \in \mathbb{R}$. Define the *incidence matrix*, $I_{\mathcal{H}}$, of size $|V| \times |E|$ for hypergraph \mathcal{H} by

$$(I_{\mathcal{H}})_{v,e} = \begin{cases} +1 & \text{if } v \in V_H(e), \\ -1 & \text{if } v \in V_T(e), \\ 0 & \text{otherwise.} \end{cases} \quad (1)$$

Given the hypergraph $H = (V, E)$ define the associated *bipartite graph* $G' = (V', E')$ where V' is partitioned into $V' = V'_L \cup V'_R$ with $V'_L = V$, $V'_R = E$ and $E' = \{(v, e) \mid v \in V \wedge v \in V_T(e)\} \cup \{(e, v) \mid e \in E \wedge v \in V_H(e)\}$. We now augment the bipartite graph G' as follows to form a new (non-bipartite) graph G'' . For each node, e , of G' corresponding to a hyperedge in V'_R replace that node by two nodes e^- and e^+ together with a directed edge (e^-, e^+) and attach the cost to that edge given by the cost $\gamma(e)$ of the

hyperedge e in the original hypergraph H . We attribute zero costs to all remaining edges in G'' .

Example: $H = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{e_1, e_2, e_3\}$ where $e_1 = (\{v_1\}, \{v_2, v_3\})$, $e_2 = (\{v_3\}, \{v_1, v_4, v_5\})$ and $e_3 = (\{v_3\}, \{v_4\})$. The incidence matrix and the graphs G' and G'' are as shown in Figure 1.

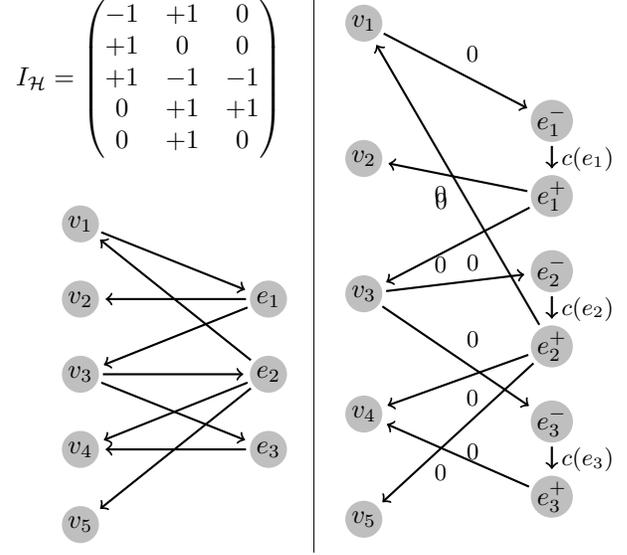


Fig. 1. **Left:** Example of a hypergraph incidence matrix $I_{\mathcal{H}}$ and its corresponding bipartite graph representation G' . **Right:** The graph G'' derived from augmenting the bipartite G' and with the costs for the hyperedges.

B. Directed Hypergraph Models for Hybrid Networks

A natural way of modeling hybrid networks with edge costs is using a directed hypergraph with weights on its hyperedges. A MANET/cellular hybrid wireless network consists of edges that belong to the following categories:

1) *Cellular point-to-point edges:* In our model, any (C,C) or (C,H) node pairs may have connectivity since the cellular base station network is assumed to be connected via wireline networks. The communication cost of a cellular link is a function of its quality. This is typically an *edge cost* which depends on the position of the endpoints of the link with respect to their nearest base stations. In this paper, we use the following cost function for a cellular edge denoted by (u, v) :

$$c_{uv} = \beta_u d(u, BS_u)^{\alpha_c} + \beta_v d(v, BS_v)^{\alpha_c}, \quad (2)$$

for some node-specific constants β_u, β_v and a path loss exponent $\alpha_c \in [2, 5]$. Here $d(u, BS_u)$ is the physical distance between u and its nearest base station node BS_u . Ideally, the choice of β_u 's will depend on the actual real cellular deployment data but here we assume $\beta_u = 1$ for all cellular/hybrid nodes u . For each occurrence of a cellular edge we construct a directed hyperedge e and assign it a cost $\gamma(e) = c_{uv}$.

2) *MANET broadcast edges:* Each MANET broadcast can be performed by a MANET (M) node or a hybrid (H) gateway node, and can simultaneously reach all its M neighbors within a fixed radio transmission range R . If a MANET node u 's

broadcast reaches one or more recipients, say v_1, v_2, \dots, v_k , we construct a directed hyperedge $e = \{u \rightarrow v_1, v_2, \dots, v_k\}$ and assign it a cost $\gamma(e) = R_m^\alpha$, where α_m is the pathloss exponent for MANETs. Note that typically $\alpha_m \geq \alpha_c$ since MANET nodes are located closer to the ground.

In Section IV-A, we use the aforementioned construction and give an approximation algorithm for computing the optimal Steiner hypergraph for solving the hybrid network multicast problem.

C. Multigraph Models for Hybrid networks

We consider now the general version of the hybrid network multicast problem where there are not only costs on the cellular point-to-point links (C-C, C-H, H-H), but also potentially on the nodes themselves. To tackle this scenario, we propose to model the hybrid network as a *multigraph*. Essentially, for all links (u, v) , if both u, v are of type H, we construct *two* edges between them in the multigraph. Such a multigraph can then be augmented appropriately to construct a simple graph as illustrated in Figure 2.

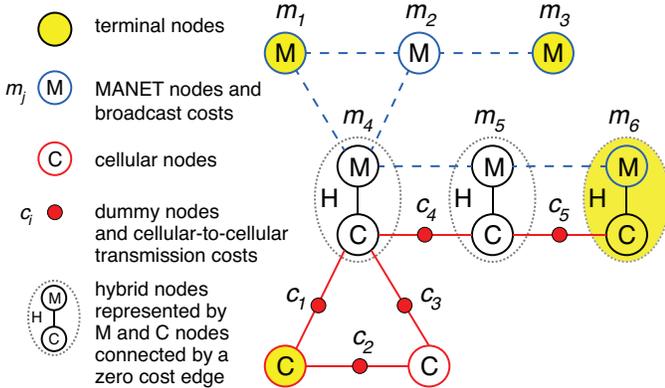


Fig. 2. An example of a multigraph representation of a hybrid network converted to a simple graph by introduction of dummy vertices for cellular edges and splitting of hybrid nodes into MANET and cellular nodes connected by zero cost edges.

In Section IV-B we give another approximation algorithm which works with the multigraph input instead of the hypergraph input. The advantage of the multigraph input is that it can be converted to a simple undirected node weighted graph with zero edge costs (cellular edges are converted to dummy nodes and edge weights are transferred to them). This construction allows us to combine techniques from Connected Dominating Sets and Node Weighted Steiner Trees for solving the hybrid network multicast problem.

IV. ALGORITHMS FOR HYBRID NETWORK MULTICAST

A. Hypergraph case

For this case, we first use the technique of transforming a given edge-weighted hybrid wireless multigraph into a directed hypergraph (see Section III). We then use a standard method to transform this directed hypergraph into a bipartite graph with costs on nodes that correspond to hyperedges in the original.

Algorithm 1 HYB-DIRECTED-MULTICAST $(H, \mathcal{M}, c(\cdot))$

- 1: Generate a bipartite graph G' from H following the steps in Section III-A. The nodes to the right side of G' correspond to hyperedges, and have associated costs.
 - 2: Generate an edge-weighted non-bipartite graph G'' from G' following the steps in Section III-A. All nodes in G'' have zero cost.
 - 3: $\mathcal{R} = \text{ROOT}(\mathcal{M})$ {One of the terminals must be a root.}
 - 4: $H_T \leftarrow \text{DIRECTEDSTEINERTREE}(G'', \mathcal{R}, \mathcal{M}, \lfloor \log_2 |\mathcal{M}| \rfloor + 1, |\mathcal{M}|)$.
 - 5: **return** H_T {Approx. optimal hybrid multicast tree.}
-

Finally, we further augment the obtained bipartite graph into a directed graph by representing each nonnegative cost node by a directed edge and transfer the node costs onto newly created edges (as in Figure 1). The latter graph serves as an input to Charikar et al.'s algorithm [4]. This algorithm represents a recursive procedure *DirectedSteinerTree* (G, r, X, i, k) which when called with $i = \lfloor \log_2 k \rfloor + 1$, where k is the size of the set of terminals X to be reached from a root r , represents an $O(\log_2 k)$ approximation algorithm to the directed Steiner tree problem and runs in quasi-polynomial time. We summarise the hypergraph approach in Algorithm HYB-DIRECTED-MULTICAST (Algorithm 1) and claim the following result. We direct the reader to a longer technical report [2] for detailed proofs of this and subsequent theorems.

Theorem 1. *Algorithm HYB-DIRECTED-MULTICAST yields an $O(|\mathcal{M}|^\epsilon)$ approximation algorithm for $\epsilon > 0$.*

B. Wireless Multigraph case

For this case, we first use the technique from Section III for transforming a given hybrid wireless multigraph to a simple node-weighted graph. After constructing such a graph, we propose to use Algorithm 2 in order to approximately find the lowest cost multicast subgraph. Algorithm 2 makes use of the following standard notation: $N(v)$ is v 's neighborhood and $N[v] = N(v) \cup \{v\}$ is v 's *closed* neighborhood; $\text{Leaf}(H)$ denotes the set of leaf nodes of a tree H . The main intuition behind Algorithm 2 is that it attempts to make use of wireless broadcast as much as possible and then “stitches together” broadcast islands by unicast paths over the cellular networks.

Theorem 2. *Consider unweighted nodes. Algorithm HYB-MULTICAST (Algorithm 2) is a polynomial-time $(3.5 \ln |\mathcal{M}| + \theta(1))$ approximation algorithm.*

Corollary 3. *Consider weighted nodes, such that the minimum weight is 1 and maximum weight is w_{\max} . Algorithm HYB-MULTICAST is a polynomial-time $w_{\max} \cdot (3.5 \ln |\mathcal{M}| + \theta(1))$ approximation algorithm.*

Theorem 4. *Consider unweighted nodes. Algorithm HYB-MULTICAST2 (Algorithm 3) is a polynomial-time $(2 \ln |\mathcal{M}| + \theta(1))$ approximation algorithm.*

Let $c_u(v)$ and $c_m(v)$ be the cost of a given node v doing a

Algorithm 2 HYB-MULTICAST ($G, \mathcal{M}, c(\cdot)$)

- 1: $G' \leftarrow G$ {Simple graph from multigraph as in Figure 2}
 - 2: $\mathcal{B}_{\mathcal{M}} \leftarrow \mathcal{M} \cap (\mathcal{V}_{\text{manet}} \cup \mathcal{V}_{\text{hyb}})$ {MANET terminals}
 - 3: **for** $v \in \mathcal{V}_{\text{manet}} \cup \mathcal{V}_{\text{hyb}}$ **do**
 - 4: $S_v \leftarrow \mathcal{B}_{\mathcal{M}} \cap N[v]$
{Greedy construct a dominating set K of M/H terminals}
 - 5: $K \leftarrow \emptyset; C \leftarrow \emptyset$
 - 6: **repeat**
 - 7: $v = \underset{u \in \mathcal{V}_{\text{manet}} \cup \mathcal{V}_{\text{hyb}}}{\operatorname{argmin}} \frac{c(u)}{|S_u \setminus C|}$
 - 8: $C \leftarrow C \cup S_v; K \leftarrow K \cup \{v\}$
 - 9: **until** $C \supseteq \mathcal{B}_{\mathcal{M}}$
 - 10: For each node in K choose a representative node in $\mathcal{B}_{\mathcal{M}}$ adjacent to it. This set is called $R(K)$. {Guha and Khuller [6]}
 - 11: $H \leftarrow$ Node-weighted Steiner tree Algorithm of Guha and Khuller [7] run on $(G', R(K) \cup \mathcal{M} \setminus \mathcal{B}_{\mathcal{M}})$ {Connect the nodes in set $R(K)$ to cellular terminals in $\mathcal{M} \setminus \mathcal{B}_{\mathcal{M}}$ }
 - 12: Delete the nodes $\text{Leaf}(H) \setminus \mathcal{M}$ from H
 - 13: **return** $H \cup K \cup R(K)$
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Algorithm 3 HYB-MULTICAST2 ($G, \mathcal{M}, c(\cdot)$)

- 1: $G' \leftarrow G$ {Simple graph from multigraph as in Figure 2}
 - 2: For each cellular edge e in G' , replace it with a length-2 path of MANET edges (e_1, e_2)
 - 3: $T \leftarrow$ Algorithm B of Guha and Khuller [7] run on (G', \mathcal{M}) {This computes a node-weighted Steiner tree with terminals \mathcal{M} , where each node v only performs MANET broadcasts to $N_{G'}(v)$ }.}
 - 4: **return** T
-

unicast or a multicast, respectively (or infinity if v cannot do either of these). Then we can obtain the following.

Corollary 5. *Consider weighted nodes, and assume that for every node v $c_{m(v)} \leq c_{u(v)}$ and for every edge (v, v') $c_{u(v)} = c_{u(v')}$. Then the approximation guarantee of Algorithm 3 still holds.*

V. PERFORMANCE EVALUATION

In this section we study the relative performance trade-offs of the two algorithms proposed in this paper by means of numerical simulations.

The following set of steps outlines the simulation procedure. We simulate a cellular base-station network by laying down a honeycomb lattice structure within an $L \times L$ area ($L = 10$), and then generate node locations according to a 2D Poisson Point Process with the density $\lambda = 0.33$. This yields a total of n nodes where n is Poisson distributed with mean λL^2 . MANET, cellular, or hybrid capability is assigned to each node randomly according to the $p_M : p_C : p_H = 7 : 1 : 2$ ratio. M - M , M - H , H - M , and H - H broadcast links are created if the corresponding nodes are located within the corresponding MANET transmission radius $R = 1.9$; cost R^{α_m} is assigned to these transmissions, where $\alpha_m = 4$. These costs are assigned

to hyperedges for Algorithm 1 and to nodes for Algorithm 2. C - C , C - H , C - H , H - H point-to-point links are created for all such pairs and costs are assigned to these links following the procedure from Section III-B. A set M of terminal nodes is chosen randomly in the largest connected component of the resultant hypergraph or multigraph. Algorithms 1 and 2 are executed on this random network.

Figure 3 plots the relative performance of Algorithms 1 and 2 as the fraction of terminal nodes $\frac{|M|}{n}$ is varied from 0 to $\frac{1}{2}$. We can observe from Figure 3(a) that Algorithm 1 tends to outperform Algorithm 2, and the gap between them generally widens as $|M|$ increases for a fixed n . But, it can be observed from Figure 3(b) that Algorithm 2 has an advantage in time complexity (and definitely space complexity) over Algorithm 1 in the range $\frac{|M|}{n} \in (0, 1/4]$. However, for $\frac{|M|}{n} > 1/4$, Algorithm 1 has the edge with respect to time complexity as well as cost. For much larger networks though, Algorithm 1's time complexity is likely to become unmanageable (due to the $O(n^2)$ growth in network size) and hence Algorithm 2 would become the more practical choice.

VI. EXPLOITING MULTICAST IN CELLULAR NETWORKS

In this paper, we have assumed that the cellular links are point-to-point (P-to-P) because that is the current configuration in which telecom operators operate their networks. However, the 3GPP and LTE standards have proposed support for point-to-multipoint (P-to-M) links anticipating the increasing popularity of voice and video multicast in future [1]. In the proposed Multimedia Broadcast Multicast Services (MBMS) [8], it is possible for a base station node (BS) to communicate either in the P-to-P mode or in the P-to-M mode to reach one or more mobiles in its own cell. These cellular P-to-M links can be incorporated into our model and technique proposed in Section III without much difficulty in the following manner.

First of all, MBMS considers the situation where the content provider is on an Internet server and the content does not originate on a mobile device—the case we have considered in this paper, as this scenario can be easily addressed by adding a P-to-P *uplink* from a mobile to a content server node (CS) and a P-to-M *downlink* from S to all the terminals in a particular cell. The aforementioned style of P-to-M cellular transmission can be modeled in our hybrid network multicast setup by making the following modification.

Secondly, instead of all mobiles with type C/H in the hybrid network directly forming a cellular clique with edge weights (with no explicit BS nodes), we now add a base station node BS_k in each cell k , an uplink from each of the C/H nodes in the cell k to BS_k , and a directed hyperedge from BS_k to all the C/H nodes in cell k . These BS nodes are assigned type BS in addition to the currently existing nodes of types C, M, or H. Now, instead of the C/H nodes forming a clique, only the BS nodes form a clique among themselves (with very low edge costs) and the C/H nodes are only connected to their nearest BS nodes in a star topology (considering only cellular links). The uplinks and P-to-P downlinks get costs $\sim \beta \cdot d(u, BS_u)^{\alpha_c}$; however, the P-to-M downlinks (which are now hyperedges)

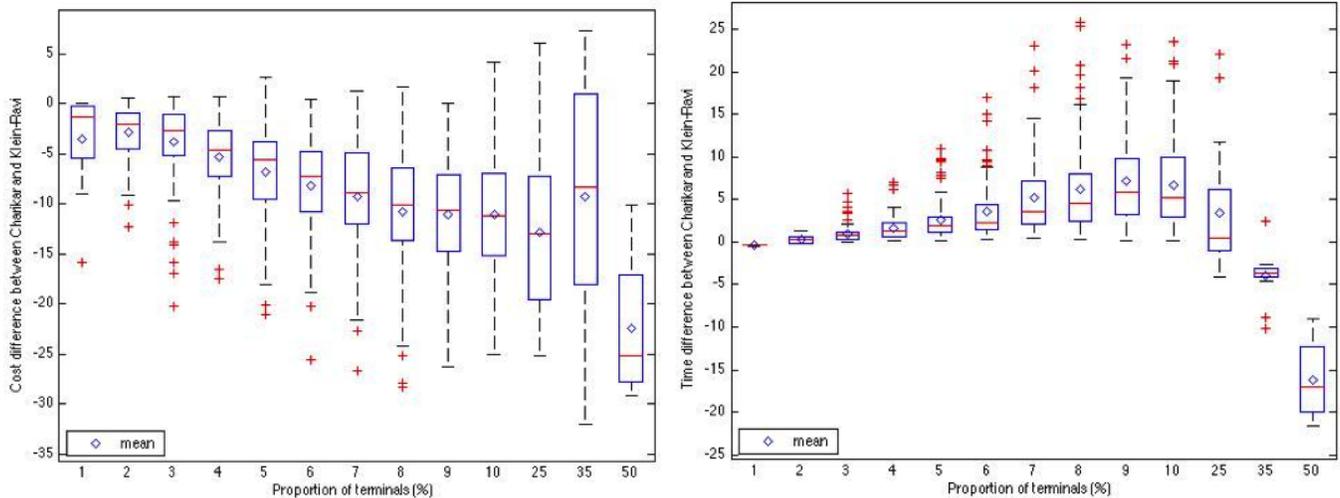


Fig. 3. Relative performance comparison between Algorithms 1 and 2 with a varying fraction of terminal nodes over multiple runs: (a) difference in costs of the multicast tree; (b) difference in running times.

get a different cost which is a function of the largest distance between a BS and a mobile in that corresponding cell. This is analogous to the MANET broadcast cost which is a function of the “transmission range”. Note that even the BS nodes may be sources of content if the content is originating somewhere in the wide area Internet.

It is easy to see that our directed hypergraph construction approach in Sections III-A and III-B and the algorithm in Section IV-A are general enough to incorporate the above modifications to the cellular network connectivity. However, the algorithm in Section IV-B needs more significant modifications. This is a topic of future research.

We note that the broadcast nature of the cellular BS has subtly different ramifications from the broadcast nature of MANET. In MANET every node can broadcast, whereas in Cellular only BS nodes can broadcast, and thus two adjacent cells in cellular can only be connected by Internet connections between corresponding BS nodes unlike in MANET where two neighborhoods can be bridged directly (and wirelessly) by intermediate MANET nodes.

VII. CONCLUDING REMARKS

In this paper, we have investigated the problem of multicasting in hybrid wireless networks possessing both point-to-point and broadcast links. We have modeled the problem with directed hypergraphs and multigraphs and gave two distinct approximation algorithms. We have showed using numerical simulations that the algorithm based on hypergraphs has a better approximation factor but a much higher time complexity than the algorithm based on multigraphs. Hence the multigraph algorithm may in practice be a feasible choice. It is also amenable to a simpler distributed implementation for dynamic networks, which we plan to investigate in the future. We plan to analytically study phase transitions and trade-off in connectivity and multicast cost using the two

algorithms proposed. We also intend to explore applications to real deployment scenarios in our future work.

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