Game Theory and Linear Programming (MAT 347/789 / CMP 464/788, 4cr)

Fall 2014, MW 6:00-7:40pm, Lehman College, Prof. M.P. Johnson

$$\max_{\mathbf{x}} \min_{\mathbf{y}} \mathbf{y}^\intercal \mathbf{A} \mathbf{x} \;=\; \min_{\mathbf{y}} \max_{\mathbf{x}} \mathbf{y}^\intercal \mathbf{A} \mathbf{x}$$

Graduate Center CS PhD students and other CUNY students are encouraged to register by ePermit.
Prerequisites: Discrete math, familiarity with linear algebra, at least one semester of calculus, and experience and comfort with writing mathematical proofs ("mathematical maturity").

Texts: *Linear Programming: Foundations and Extensions* (Vanderbei; PDF) and *Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations* (Shoham and Leyton-Brown; PDF)

Assessment: Several bi-weekly problem sets, midterm, and final. Also, participation. The problem sets will be something like 85% mathematical and 15% AMPL implementation.

Course summary: This is an advanced (junior/senior/graduate-level) *theory* course (meaning, primarily math) in game theory and linear programming, paying special attention to the aspects of these two subjects that relate to one another: in linear programming—*the mathematical/computational framework used to solve large-scale optimization problems in business, government, and elsewhere*—we will emphasize the geometric view and the nature of LP duality; in game theory—*the mathematical theory of the choices made by selfish agents with competing interests*—we will emphasize zero-sum 2-person games, the minimax theorem, and the computation of equilibria.

Time permitting, we will also explore some further topics from game theory including the complexity of computing Nash equilibria, Bayesian games, auctions and mechanism design, and voting and social choice. Although primarily mathematical, this course will also have a "programming" component: we will regularly implement LPs in AMPL and solve them (using, e.g., NEOS).

Finally, please note—*please* note—that game theory has nothing to do with video games and computer graphics and whatnot. Game theory does, however, have lots of real, non-fanciful applications in settings like economics, politics, and life: classical problems like tragedy of the commons and prisoner's dilemma (or in the Cold War context, *mutually assured destruction*), collective action problems, perverse incentives and negative externalities and Pigovian taxes, etc., etc.

Course topics:

- *linear programming* fundamentals: polyhedra and the geometry of LPs, extreme points, basic solutions, the simplex method, convex sets, LP duality, complementary slackness
- *AMPL* and example applications (diet problem, portfolio optimization) and *integer programming* (column generation, branch and bound)
- *combinatorial problems as LPs* (max-flow/min-cut, transportation problem, assignment problem) and the *primal-dual method*
- *game theory* fundamentals: matrix games, optimal strategies, 2-person zero-sum games, pure and mixed strategies, solving games by LP, Nash equilibria, the minimax theorem and LP duality, normal form, extensive form, backward induction, subgame perfect equilibrium
- further topics in game theory: *complexity* of computing Nash equilibria (PPAD, etc.), Bayesian games & Bayes-Nash equilibrium, *auctions and mechanism design* (price of anarchy, Vickrey and VCG, incentive compatibility, Myerson's lemma), *voting and social choice* (Arrow's & Gibbard-Satterthwaite's impossibility theorems)