New Functions from Old Functions

We will approach this topic by providing examples and plots of functions. First, we will demonstrate some definitions and related Maple syntax by establishing some equations in a worksheet. The reader might benefit by typing in each line or copying and pasting the commands from **C1Notes** as found on the syllabus webpage that the instructor provided.

```
>  restart:
>  f:=x->x^2-x+1;    g:=x->sin(2*x);
```
$$f := x \rightarrow x^2 - x + 1$$
$$g := x \rightarrow \sin(2x)$$

Addition
```
>  (f+g)(x)=f(x)+g(x);
```
$$x^2 - x + 1 + \sin(2x) = x^2 - x + 1 + \sin(2x)$$
```
>  plot([f(x),g(x),f(x)+g(x)],x=-1..2,color=[red,green,blue]);
```
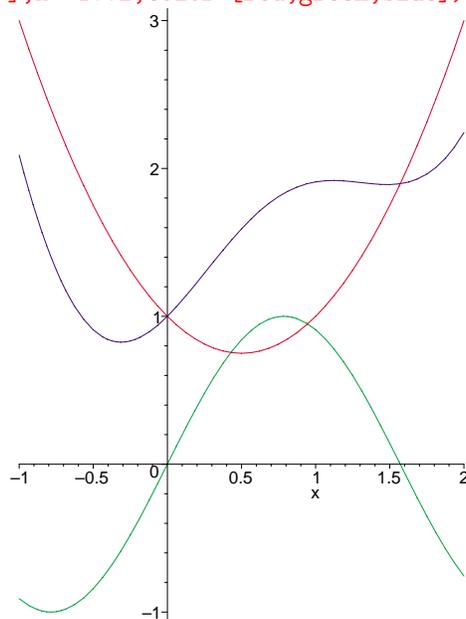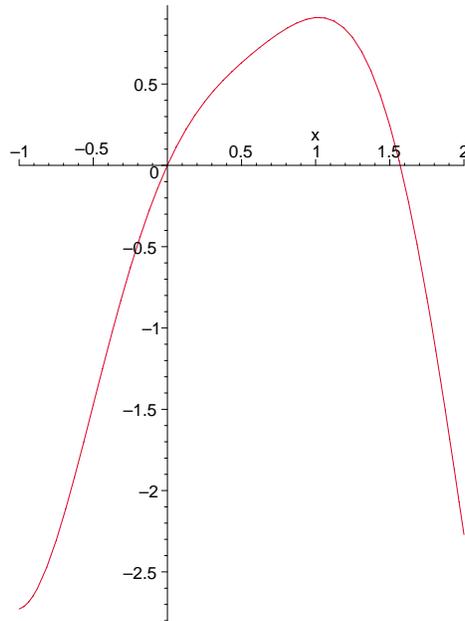

Subtraction
```
>  (f-g)(x)=f(x)-g(x);
```
$$x^2 - x + 1 - \sin(2x) = x^2 - x + 1 - \sin(2x)$$

Multiplication
```
>  (f*g)(x)=f(x)*g(x);
```
$$(x^2 - x + 1)\sin(2x) = (x^2 - x + 1)\sin(2x)$$
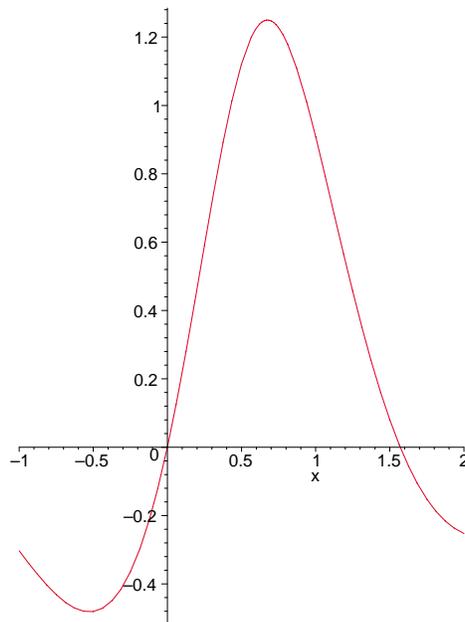```
> plot((f*g)(x),x=-1..2);
```

**Division**
```
> (g/f)(x)=g(x)/f(x);
```

$$\frac{\sin(2x)}{x^2 - x + 1} = \frac{\sin(2x)}{x^2 - x + 1}$$
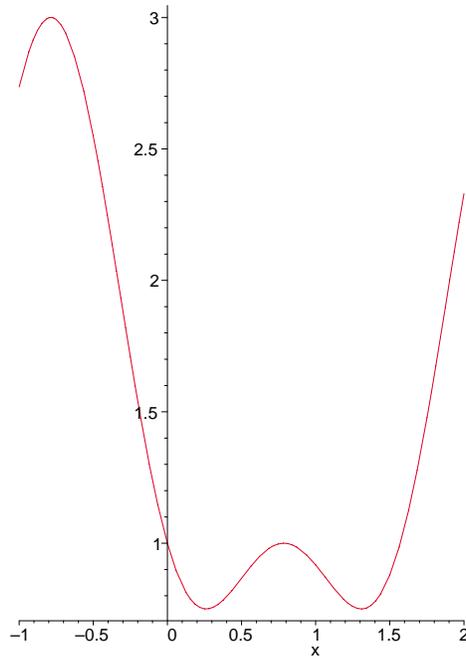
```
> plot((g/f)(x),x=-1..2);
```



**Composition**
```
> (f@g)(x)=f(g(x));
```

$$\sin(2x)^2 - \sin(2x) + 1 = \sin(2x)^2 - \sin(2x) + 1$$
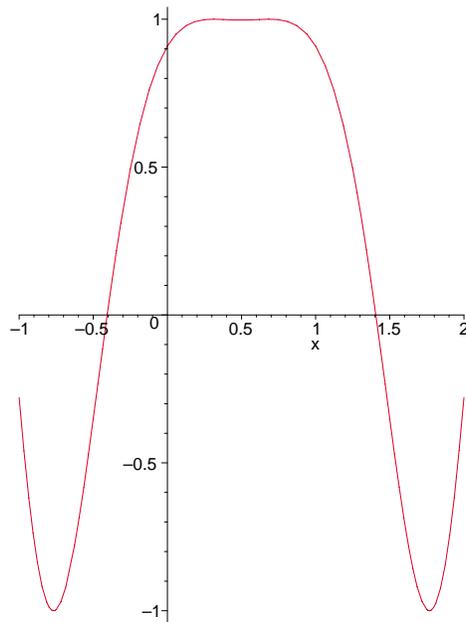
```
> plot((f@g)(x),x=-1..2);
```

**Composition**

```
>  (g@f)(x)=g(f(x));
```

$$\sin(2x^2 - 2x + 2) = \sin(2x^2 - 2x + 2)$$

```
>  plot((g@f)(x),x=-1..2);
```


Maple Example   For the function $f(x) = x + \sin(x)$, find all values of x for which $f(x + \pi/6) = f(x - \pi/6) + 1$.

```
>  f:=x->x+sin(x);
```

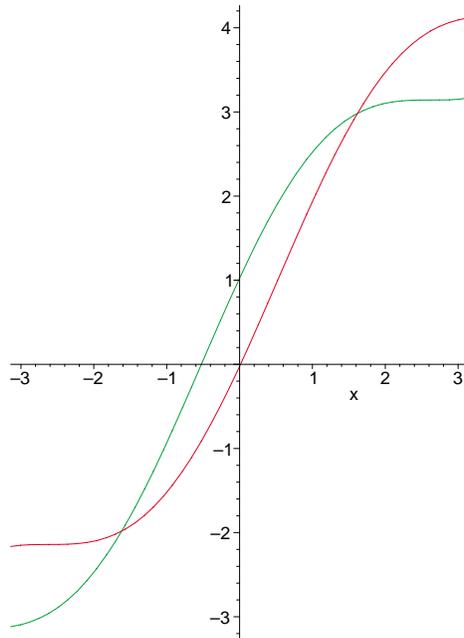$$f := x \longrightarrow x + \sin(x)$$

```
>  eq1:=f(x-Pi/6)+1=f(x+Pi/6);
```

$$eq1 := x - \frac{\pi}{6} - \cos\left(x + \frac{\pi}{3}\right) + 1 = x + \frac{\pi}{6} + \sin\left(x + \frac{\pi}{6}\right)$$

```
>  evalf(solve(eq1,x));
```

$$-1.618011419, \, 1.618011418$$

```
>  plot([f(x-Pi/6)+1,f(x+Pi/6)],x=-Pi..Pi);
```

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     We see from the graph that the curves cross at two points, so we found all of the solutions using Maple. This is not always the case, especially when there are an infinite number of solutions. There is another Maple command, **fsolve**, that returns a floating point number and will permit one to limit the range where a solution is being sought. This next example illustrates this.

Maple Example: Suppose that $h(x) = -\frac{x}{2} + 2$ and $j(x) = \sin(x)$, find all solutions of $h(x) = 2\,j(x)$.

> `h:=x->-x/2+2;  j:=x->sin(x);`

$$h := \to -\frac{x}{2} + 2$$
$$j := \sin$$
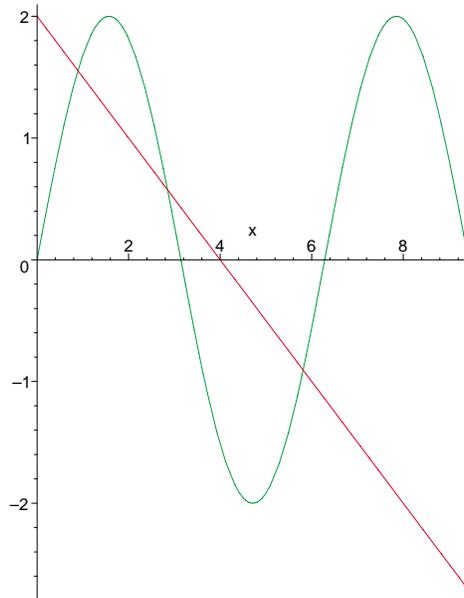
> `solve(h(x)=2*j(x),x);`

$$\text{RootOf}(\_Z - 4 + 4\sin(\_Z))$$

> `evalf(%);`

$$.8904870807$$

By using **evalf** we get a numerical answer for our solution. But, we suspect that other solutions might exist so we plot the graphs of each side of our equation. We see that there are two more solutions, one between 2 and 4, the other between 4 and 6.2. The number 6.2 is slightly larger than 2π, where $\sin(x)$ has

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a zero and the intersection is obviously before that. We turn to `fsolve` and select a range for the value, which `fsolve` permits. However, `fsolve` seeks a numerical solution and returns a single floating point number, not more.

```
>  plot([h(x),2*j(x)],x=0..3*Pi);
```

```
>  fsolve({h(x)=2*j(x)},{x},2..4);
```
$$\{x = 2.849968934\}$$
```
>  fsolve({h(x)=2*j(x)},{x},4..(6.2));
```
$$\{x = 5.812826090\}$$

Note that when we use set notation in `evalf`, Maple returns the output in similar notation.

C1M1 Problems: Use Maple to plot the graphs to see where they intersect, and to find the solutions.

1. For $f(x) = \frac{x}{2} + 2$ and $g(x) = 2^x$, find all solutions of $f(x) = g(f(x-2))$.

2. For $p(x) = \sin(x/6)$ and $q(x) = \sqrt{x^2 + 4}$, find all solutions of $p(q(x)) - x^2 = x$ for $-2 \le x \le 2$.

3. For $r(x) = \cos(x)$, find all solutions of $(r \circ r)(x) = x$.