C2M7

Solutions of Differential Equations

A differential equation arises when there is a relationship involving a function and one or more of its derivatives. For example

$$y \quad + 5y \quad + 6y = 0$$

is such an equation. A function is a solution of this equation if you obtain 0 when you add its second derivative to 5 times its first derivative and then add 6 times the function itself.

Maple Example 1 Use Maple to verify that $y(t) = ae^{-3t} + be^{-2t}$ is a solution of the differential equation shown above, where a and b are arbitrary constants.

```
> with(student):
> de1:={diff(y(t),t,t)+5*diff(y(t),t)+6*y(t)=0};
```

$$de1 := \{\left(\frac{\partial^2}{\partial t^2} y(t)\right) + 5\left(\frac{\partial}{\partial t} y(t)\right) + 6\,y(t) = 0\}$$

```
> y1:=a*exp(-3*t)+b*exp(-2*t);
```

$$y1 := a\mathbf{e}^{(-3t)} + b\mathbf{e}^{(-2t)}$$

```
> eval(de1,y(t)=y1);
```

$$\{0 = 0\}$$

which shows that for any constants a and b, $y(t)$ is a solution of the given equation.

Maple Example 2 Determine whether $y(x) = e^x + ce^{-2x}$ is a solution of

$$y \quad + 2y = 3e^x$$

for any value of the constant c.

```
> de2:={diff(y(x),x)+2*y(x)=3*exp(x)};
```

$$de2 : \{\left(\frac{\partial}{\partial x} y(x)\right) + 2y(x) = 3\mathbf{e}^x\}$$

```
> y2:=exp(x)+c*exp(-2*x);
```

$$y2 := \mathbf{e}^x + c\mathbf{e}^{(-2x)}$$

```
> eval(de2,y(x)=y2);
```

$$\{3\mathbf{e}^x = 3\mathbf{e}^x\}$$

How would we know if we did not have a solution? let's define a different function and see what happens.

```
> y3:=2*exp(x)+C*exp(-2*x);
```

$$y3 := 2\mathbf{e}^x + C\mathbf{e}^{(-2x)}$$

```
> eval(de2,y(x)=y3);
```

$$\{6\mathbf{e}^x = 3\mathbf{e}^x\}$$

Now in order for $y3$ to be a solution, the last equation, $6e^x = 3e^x$, would have to be true for every x. But this is true for *no* x, so $y3$ is not a solution.

C2M7 Problems: Use Maple and the method illustrated above to determine whether the given function is a solution of the differential equation.

1. $y = \sin x + x^2$, $\qquad y \quad + y = x^2 + 2$
2. $y = e^{2x} - 3e^{-x}$, $\qquad y \quad - y \quad - 2y = 0$
3. $x = 2e^{3t} - e^{2t}$, $\qquad \frac{d^2x}{dt^2} - x\frac{dx}{dt} + 3x = -2e^{2t}$
4. $x = \cos 2t$, $\qquad \frac{dx}{dt} + tx = \sin 2t$
5. $x = \cos t - 2\sin t$, $\qquad x \quad + x = 0$