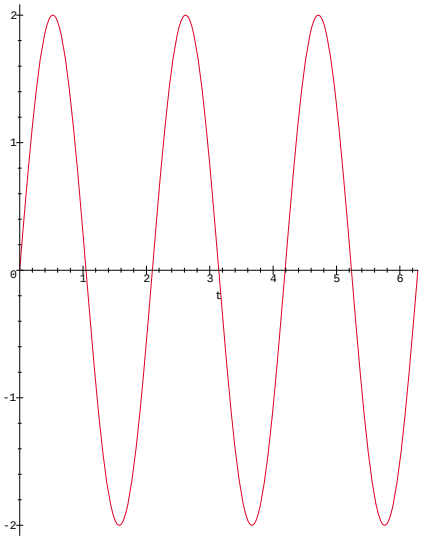
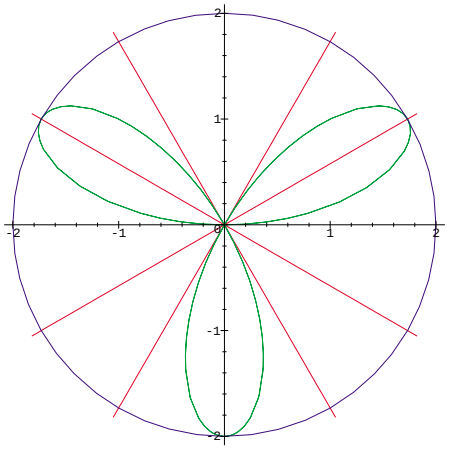
C2M6

Polar Coordinates

For many graphs in polar coordinates the easiest place to start is with a rectangular plot. We will call these *helper diagrams*. For the function $r = 2\sin(3\theta)$ we have the rectangular plot on the left and the polar plot on the right. We can see from the helper diagram that as θ varies from 0 to $\pi/3$ the radius will start at 0, reach 1 at $\pi/6$, and decrease back to 0 at $\pi/3$. The next thing we observe is that from $\pi/3$ to $2\pi/3$ the radius will take on negative values. At each point where the helper diagram attains the value 2, the polar graph will be two units from the origin for that angle. And, whenever the helper diagram has the value of -2, the polar graph will be two units in the opposite direction of that angle. At points on the helper diagram where the value is 0, the polar graph will be tangent to that angle as the graph passes through the polar origin. To sketch the polar plot, begin by drawing a circle of radius 2, then draw radial lines for multiples of $\pi/6$. Mark the intersection for each peak and valley of the helper diagram. These have polar coordinates whose angles correspond to odd multiples of $\pi/6$. Remember, sometimes it is a 2 and sometimes it is a radius of -2. On the polar graph, put your finger at the origin and trace the plot, realizing that you are going to go around the plot twice.

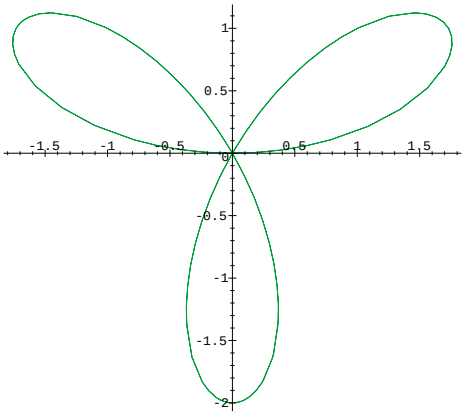


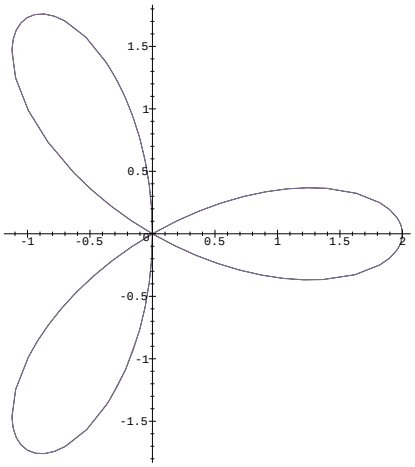


Plotting in polar coordinates is very easy using Maple. We need the library `plots` in order to use `polarplot`. We will use t instead of θ as the variable sometimes.

Maple Example 1: Plot $r = 2\sin(3\theta)$. On the right, $r = 2\cos(3\theta)$ is displayed so that you may compare.

```
> with(plots):
> polarplot(2*sin(3*t),t=0..2*Pi,color=green);
```

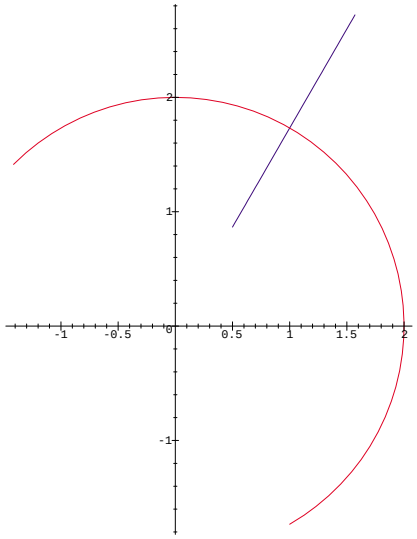




Maple Example 2: Polarplot may also be used parametrically. We will define two plots and give them names, *A1* and *B1*, and end their lines with colons to suppress the output. Using `polarplot` parametrically, the first coordinate determines r and the second determines θ. Each may be functions of a third variable whose domain must be specified within the square brackets.

Maple Example 3:

```
> with(plots):
> A1:=polarplot([r,Pi/3,r=1..Pi],color=blue):
> B1:=polarplot([2,theta,theta=-Pi/3..3*Pi/4]):
> display({A1,B1});
```

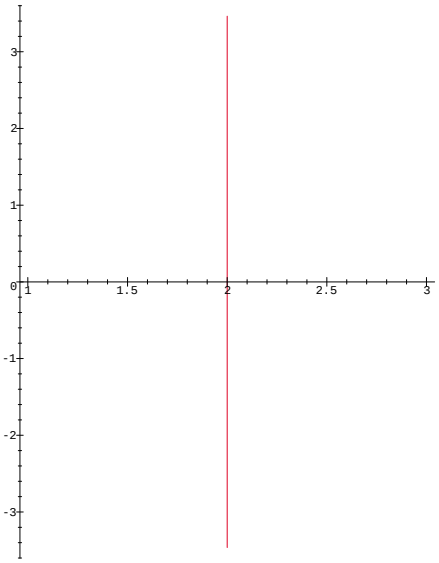


Maple Example 4: Suppose we consider the vertical line $x = 2$. We know that $x = r\cos(\theta)$ in polar coordinates, so set the x values equal. This means that $r\cos(\theta) = 2$ and that we may solve for r, which produces

$$r = \frac{2}{\cos(\theta)} = 2\sec(\theta)$$

We must avoid dividing by 0, so odd multiples of $\pi/2$ must be avoided. Here we will stay between $-\pi/3$ and $\pi/3$.

```
> with(plots):
> polarplot([2*sec(t),t,t=-Pi/3..Pi/3]);
```



C2M6 Problems Use Maple to determine the polar graphs of the given functions. Remember, you must set the domain. Also, resize the output to a reasonable size. Save paper.

1. $r = 1 + \cos(\theta)$ 2. $r = 2\csc(\theta)$ 3. $r = \sqrt{2} - 2\cos(\theta)$ 4. $r = \theta,\ -\pi \leq \theta \leq 2\pi$