C2M4

Improper Integrals

Improper integrals can occur in two different ways. The interval of integration can be unbounded, or the integrand can be an unbounded function. As you know, in the respective cases

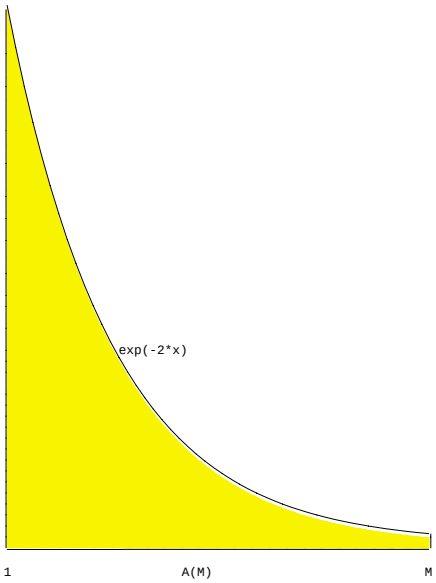
$$\int_a^{\infty} f(x)\,dx \equiv \lim_{M\to\infty} \int_a^{M} f(x)\,dx$$

$$\int_a^{b} f(x)\,dx \equiv \lim_{t\to a^+} \int_t^{b} f(x)\,dx$$

where, in the second case f is unbounded at a. In Maple, we will evaluate the integral from a to M and then evaluate the limit of that result as the definition suggests in order to reinforce the concepts.

Maple Example:

Evaluate the improper integral $\int_1^{\infty} e^{-2x}\,dx$



We find an expression $A(M)$ which expresses the area of the region under the curve as a function of the righthand endpoint, M. Then we evaluate the limit

$$\lim_{M\to\infty} A(M)$$

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> with(student):
> A:=Int(exp(-2*x),x=1..M);
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$$A := \int_1^{M} e^{(-2x)}\,dx$$

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> A:=value(A);
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$$A := -\frac{1}{2}e^{(-2M)} + \frac{1}{2}e^{(-2)}$$

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> limit(A,M=infinity);
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$$\frac{1}{2}e^{(-2)}$$

Maple Example: Evaluate the improper integral $\int_0^1 \frac{1}{x^{1/3}}\,dx$ Note that the integrand is discontinuous at the lefthand endpoint.

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> B:=Int(x^(-1/3),x=t..1);
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$$B := \int_t^{1} \frac{1}{x^{1/3}}\,dx$$

```
> B:=value(B);
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$$B := \frac{3}{2} - \frac{3}{2}t^{2/3}$$

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> limit(B,t=0);
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$$\frac{3}{2}$$

C2M4 Problems Evaluate the improper integrals using Maple.

1. $\displaystyle\int_2^\infty \frac{1}{x^{4/3}}\,dx$

2. $\displaystyle\int_3^\infty \frac{\ln x}{x}\,dx$

3. $\displaystyle\int_0^8 x^{-2/3}\,dx$

4. $\displaystyle\int_0^1 x\ln x\,dx$

5. $\displaystyle\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$