C2M12

Ratio Test for Power Series

We extend our use of Maple to power series by employing the same approach as in the previous section, but realizing that there will be a variable, or parameter, x involved. We may no longer assume that all the terms are positive, so the absolute value **must** be used.

Example: Find the open interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{2^n\, x^n}{n\, 3^{n+1}}$.

$\left|\dfrac{a_{n+1}}{a_n}\right| = \left|\dfrac{2^{n+1}x^{n+1}}{(n+1)\,3^{n+2}} \cdot \dfrac{n\,3^{n+1}}{2^n\,x^n}\right| = \dfrac{2}{3}\left|\dfrac{n}{n+1}\right| \cdot |x| \to \dfrac{2}{3}|x|$ with the limit taken as n goes to infinity and x is held constant. When are we guaranteed that this series will converge? When the limit of the ratio test is forced to be less than one. Thus, $\dfrac{2}{3}|x| < 1 \;\Rightarrow\; |x| < \dfrac{3}{2}$. We conclude that the series converges for all x in the open interval $\left(-\frac{3}{2}, \frac{3}{2}\right)$.

Maple Example: Use Maple to find the open interval of convergence of the previous example.

```
> an:=n->(2^n*x^n)/(n*3^(n+1));
```

$$an := n \to \frac{2^n\, x^n}{n\, 3^{(n+1)}}$$

```
> rn:=an(n+1)/an(n);
```

$$rn := \frac{2^{(n+1)}\, x^{(n+1)}\, n\, 3^{(n+1)}}{(n+1)\, 3^{(n+2)} 2^n\, x^n}$$

```
> rn:=abs(simplify(rn));
```

$$rn := \frac{2}{3}\left|\frac{xn}{n+1}\right|$$

```
> limit(rn,n=infinity);
```

$$\frac{2}{3}|x|$$

```
> solve(%<1,x);
```

$$\text{RealRange}\left(\text{Open}\left(\frac{-3}{2}\right), \text{Open}\left(\frac{3}{2}\right)\right)$$

So the open interval $(-3/2, 3/2)$ is our answer.

C2M12 Problems: Use Maple to find the open interval of convergence of the given power series.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{n!x^n}{n^n}$
2. $\displaystyle\sum_{n=1}^{\infty} \frac{nx^n}{(n+1)!}$
3. $\displaystyle\sum_{n=1}^{\infty} \frac{n!(2n)!x^n}{(3n)!}$
4. $\displaystyle\sum_{n=1}^{\infty} \frac{(2/3)^n(x+2)^n}{n^2}$