Inverting Functions with Exponentials and Logs

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A supplement for MAT172 using Larson and Hostetler Ed 6

Before doing this assignment, review Section 1.8, particularly Examples 6 and 7 on inverses. Here we extend those techniques using our new knowledge from 3.1-4.

Example: Let \( f(x) = e^{3x-1} \). What are the domain and range of \( f \)? What is the inverse of \( f \)? Be sure to state the domain and range of the inverse as well as giving us the formula.

Solution: Well the domain of \( f \) is \((-\infty, \infty)\) because there is an output for every possible input. The range is \((0, \infty)\) because all the outputs are positive.

To get a formula for \( f^{-1}(x) \), we write \( y = e^{3x-1} \) and solve for \( x \).

\[
y = e^{3x-1}
\]

\[
\ln(y) = \ln(e^{3x-1})
\]

\[
\ln(y) = 3x - 1
\]

\[
\ln(y) + 1 = 3x
\]

\[
(Ln(y) + 1)/3 = x
\]

We have solved for \( x \), which is the output of \( f^{-1} \) when the input is \( y \). So replace \( x \) by \( f^{-1}(x) \) and replace \( y \) by \( x \):

\[
f^{-1}(x) = (\ln(x) + 1)/3.
\]

Note this is NOT the same as \( \ln(x + 1)/3 \) or \( \ln((x + 1)/3) \).

The domain of \( f^{-1} \) is the range of \( f \), which is \((0, \infty)\) and that makes sense because we can take \( \ln(x) \) for any positive number \( x \).

The range of \( f^{-1} \) is the domain of \( f \), is \((-\infty, \infty)\).

Check:

\[
f(f^{-1}(x)) = f((\ln(x) + 1)/3) \text{ plugging in the formula for } f^{-1}(x) \tag{1}
\]

\[
= e^{3((\ln(x)+1)/3)-1} \text{ substituting into the formula for } f \tag{2}
\]

\[
= e^{(\ln(x)+1)-1} \text{ cancelling the threes} \tag{3}
\]

\[
= e^{\ln(x)} \text{ cancelling the ones} \tag{4}
\]

\[
= x \text{ because } \ln \text{ is the inverse of } e^x. \tag{5}
\]
Also check:

\[
f^{-1}(f(x)) = f^{-1}(e^{3x-1}) \text{ plugging in the formula for } f(x) \tag{6}
\]
\[
= (Ln(e^{3x-1}) + 1)/3 \text{ substituting into the formula for } f^{-1} \tag{7}
\]
\[
= ((3x - 1) + 1)/3 \text{ because Ln and e cancel since they are inverses} \tag{8}
\]
\[
= (3x)/3 = x \tag{9}
\]

Since \( f^{-1}(f(x)) = x \) and \( f(f^{-1}(x)) = x \) we have verified that we have the correct formula for the inverse.

Problems: (answers at the end for problems 1-5)

1. Find the domain, range and inverse for \( f(x) = e^{2x+7} \).
2. Find the domain, range and inverse of \( f(x) = Ln(x - 5) \). Warning, the Ln is only defined on positive numbers so one needs \( x - 5 > 0 \).
3. Find the domain, range and inverse of \( f(x) = 5^{x+2} \).
4. Find the domain, range and inverse of \( f(x) = Log_2(8x) \).
5. Find the domain, range and inverse of \( f(x) = e^{x} + 3 \). Be careful with the range. Think about how graphs shift.
6. Find the domain, range and inverse for \( f(x) = 2^{4x-5} \).
7. Find the domain, range and inverse of \( f(x) = Ln(x + 13) \).
8. Find the domain, range and inverse of \( f(x) = 2^{x+5} \).
9. Find the domain, range and inverse of \( f(x) = Log_5(25x) \).
10. Find the domain, range and inverse of \( f(x) = e^{2x} - 10 \).

Answers:

1. The domain of \( f \) is \(( -\infty, \infty )\) and the range is \(( 0, \infty )\). The inverse is \( f^{-1}(x) = (Ln(x) - 7)/2 \) and its domain is \(( 0, \infty )\) and its range is \(( -\infty, \infty )\).
2. The domain of \( f \) is \(( 5, \infty )\) and the range is \(( -\infty, \infty )\). The inverse is \( f^{-1}(x) = e^{x+5} \) and its domain is \(( -\infty, \infty )\) and its range is \(( 5, \infty )\).
3. The domain of \( f \) is \(( -\infty, \infty )\) and the range is \(( 0, \infty )\). The inverse is \( f^{-1}(x) = Log_5(x) - 2 \). The domain of \( f^{-1} \) is \(( 0, \infty )\) and the range is \(( -\infty, \infty )\).
4. The domain of \( f \) is \(( 0, \infty )\), the range is \(( -\infty, \infty )\), the inverse is \( f^{-1}(x) = 2^{x}/8 \) or \( = 2^{x-3} \) which is the same thing. The domain of \( f^{-1} \) is \(( -\infty, \infty )\) and the range is \(( 0, \infty )\).
5. The domain is \(( -\infty, \infty )\), the range is \(( 3, \infty )\), the inverse is \( f^{-1}(x) = Ln(x - 3) \). The inverse’s domain is \(( 3, \infty )\) which makes sense because the input for \( Ln \) must be positive, and the inverse’s range is \(( -\infty, \infty )\).