Mat 604 Sample Optional Exam

Name

Answer at least 100 points worth of questions. I will give extra credit for extra points.

1. Assuming only the axioms for counting numbers and the definition of addition, state and prove the additive associative law for counting numbers. (15 points)
2. Assuming all the properties of addition and multiplication for counting numbers define x^y. Prove that for any counting numbers x, y, z $x^{y+z} = x^y \cdot x^z$. (15 points)
3. State the definition of $x > y$ for counting numbers. Prove that this relation between counting numbers is transitive. (10 points)
4. How would you define the least member of a set. State the "well ordering theorem" for the counting numbers. (10 points)
5. (a) Write out the addition table and multiplication table for the integers mod 6.
   (b) Write out the axioms for a field. Do the integers mod 6 satisfy these axioms?
   (15 points)
6. (a) Use the Euclidean algorithm to show that the greatest common divisor of 118 and 79 is 1 and then find a neighbor.
   (b) Using part a, find the continued fraction expansion of 118/79.
   (15 points)
7. (a) Draw the Farey Diagram and locate 7/8.
   (b) Use the diagram to find the continued fraction expansion for 7/8.
   (15 points)
8. State an axiom that can be used to define *completeness* for a field. (15 points)
9. How do you know what value to give the infinite continued fraction $[1, 1, 1, \ldots,]$ (10 points)
10. If $z = x + iy$ is a complex number such that $x^2 + y^2 = 1$ show that if $\bar{z} = x - iy$ then $z\bar{z} = 1$. (10 points)