

Name

Answer at least 100 points worth of questions. I will give extra credit for extra points.

1. Assuming only the axioms for counting numbers and the definition of addition, state and prove the associative law for counting numbers. (10 points)
2. State the definition of $x > y$ for counting numbers. Prove that this relation between counting numbers is transitive. (10 points)
3. (15 points)
 - (a) Write out the addition table and multiplication table for the integers mod 5.
 - (b) Write out the axioms for a field. Do the integers mod 5 satisfy these axioms?
4. Use the “9’s complement” subtraction algorithm compute (base 10) $528 - 206$. (10 points)
5. (a) Use the Euclidean algorithm to find the greatest common divisor of 107 and 89.
(b) Find the continued fraction expansion of $107/89$ from part a.
(15 points)
6. (a) Draw the Farey Diagram and locate $5/8$.
(b) Use the diagram to find the continued fraction expansion for $5/8$.
(15 points)
7. State an axiom that can be used to define completeness for a field. (15 points)
8. Show that the sequence $a_n = 1/n^2$ is a Cauchy sequence. What is its limit? (15 points)
9. If $z = x + iy$ is a complex number such that $x^2 + y^2 \neq 0$ find a complex number z' such that $zz' = 1$. (10 points)