Mat 604 Sample Optional Exam

Name

1. Assuming only the axioms for counting numbers and the definition of addition, state and prove the associative law for counting numbers. (10 points)

2. State the definition of $x > y$ for counting numbers. Prove that this relation between counting numbers is transitive. (10 points)

3. State the well ordering theorem for whole numbers. (10 points)

4. (a) State the definition of zero (that is, define it by how it behaves with respect to addition, multiplication and exponentiation.

   (b) Assuming the properties of the counting numbers prove that $x^{y+z} = x^y \cdot x^z$ holds for x a counting number and y, z whole numbers.

   (10 points)

5. Write out the addition table for the digits base 5. (10 points)

6. Use the "9's complement" subtraction algorithm compute (base 10) $528 - 206$. (10 points)

7. State the definition of the relation $\frac{a}{b} \sim \frac{c}{d}$ that defines rational numbers. Prove that multiplication of rationals is commutative. (10 points)

8. (a) Use the Euclidean algorithm to find the greatest common divisor of 107 and 89.

   (b) Find the continued fraction expansion of 107/89 from part a.

   (15 points)

9. (a) Draw the Farey Diagram and locate 5/8.

   (b) Use the diagram to find the continued fraction expansion for 5/8.

   (15 points)