Mat 604 Sample Final Exam

Name

1. Assuming the axioms for counting numbers, the definitions of multiplication, exponentiation and the associative and commutative laws for multiplication for the counting numbers, prove that for any counting numbers x, y and z, $(x \cdot y)^z = x^z \cdot y^z$. (10 points)

2. Let x, y, z be whole numbers with $y \geq z$.

   (a) Give the definition of $x - y$.

   (b) Prove that $(x + y) - z$ and $y - z$ are defined

   (c) Prove that $(x + y) - z = x + (y - z)$

   (10 points)

3. (a) State the definition of zero (that is, define it by how it behaves with respect to addition, multiplication and exponentiation.

   (b) Let x be a whole number and let y be a counting number. Assuming the trichotomy law, prove that $x + y \neq x$.

   (10 points)

4. Prove by induction that $x^{y+z} = x^y \cdot x^z$ holds for x, y, z a counting numbers. (10 points)

5. Write out the addition table for the digits base 5. (10 points)

6. State the definition of the relation $\frac{a}{b} \sim \frac{c}{d}$ that defines rational numbers. Prove that multiplication of rationals is associative. (10 points)

7. (a) Use the Euclidean algorithm to find the greatest common divisor of 107 and 89.

   (b) Find the continued fraction expansion of 107/89 from part a.

   (15 points)

8. (a) Draw the Farey Diagram and locate 5/8.

   (b) Use the diagram to find the continued fraction expansion for 5/8.

   (15 points)

9. If A and B are infinite decimals with $A > B$, show there is a terminating decimal T such that $A > T > B$; that is, there is a terminating decimal between any pair of infinite decimals. (10 points)