

**Name**

Answer any four of the five questions. Each counts 25 points.

1. (a) Define the greatest common divisor of two counting numbers.  
(b) Use the Euclidean Algorithm to find the greatest common divisor of 549 and 49.
2. (a) Draw enough of the Farey diagram so that you can find  $10/7$ .  
(b) When are two points neighbors?  
(c) Prove that if  $a/b$  and  $c/d$  are neighbors then  $a/b + n$  and  $b/c + n$  are also neighbors for all integers  $n$ .
3. (a) Find one neighbor for  $8/5$   
(b) Use the neighbor you found to find the rightmost and leftmost neighbors.  
(c) Use the neighbor you found to find a neighbor  $p/q$  such that  $|8/5 - p/q| < 1/100$ .  
(d) Draw a diagram of the umbrella and fan for  $8/5$ .
4. (a) Find the continued fraction for  $8/13$  by first drawing a circle centered on the real axis and crossing the vertical line at  $0/1$  that has  $8/13$  as a right endpoint and then reading off the cutting sequence of L's and R's.  
(b) Check that your answer gives the correct continued fraction.
5. Use the circle centered at  $1/2$  with radius  $\sqrt{5}$  to show that the continued fraction for the real number  $(1 + \sqrt{5})/2$  is given by  $[1, 1, 1, \dots]$ .