Mat 604 Sample Midterm

Oct 22.

Name

1. State the five axioms for the counting numbers. (20 points)
2. Define addition for counting numbers. Show by induction that for each counting number x, $1 + x = x^+$ (15 points)
3. State the definition of $x > y$ for counting numbers. Prove that for any counting numbers y and n, $y + n \neq y$. (Hint: use the fact that every counting number except 1 has a predecessor and exercise 2.) (15 points)
4. Use the well-ordering theorem for counting numbers to prove that there are no counting numbers between 1 and 2. Is the same true for integers? rationals? (15 points)
5. State the Transitivity law for counting numbers. (10 points)
6. Assuming you know everything about the whole numbers, state the definition of $x \cdot y$ for the integers $\mathbf{Z}$. Prove that $(-x) \cdot (-y) = x \cdot y$. (15 points)
7. Prove that multiplication for rationals is commutative. That is, show that

   $$[\frac{a}{b}]\,[\frac{c}{d}] = [\frac{c}{d}]\,[\frac{a}{b}]$$

   (10 points)