Mat 604 Sample Final Exam

Name

Answer at least 100 points worth of questions. I will give extra credit for extra points.

1. Assuming the properties of signed numbers, state the equivalence relation that defines rational numbers. Prove the commutative law for multiplication of rationals. (15 points)

2. State the definition of $x > y$ for fractions. Show that this is well defined for rationals by proving that the definition is independent of the choice of fractional representatives. (15 points)

3. Show that the complex numbers form a field. You may use the fact that the real numbers form a field. (15 points)

4. (a) Find the continued fraction for 8/13 by first drawing a circle centered on the real axis and crossing the vertical line at 0/1 that has 8/13 as a right endpoint and then reading off the cutting sequence of L's and R's.

   (b) Check that your answer gives the correct continued fraction.

   (15 points)

5. Use the circle centered at 1/2 with radius $\sqrt{5}$ to show that the continued fraction for the real number $(1+\sqrt{5})/2$ is given by $[1, 1, 1, \ldots]$. (15 points)

6. State the nested intervals axiom for completeness (5 points)

7. State the least upper bound axiom for completeness (5 points)

8. State the Cauchy sequence axiom for completeness (5 points)

9. State the continued fraction axiom for completeness (5 points)

10. Show that the sequence $a_n = 2n/(n+2)$ is a Cauchy sequence. What is its limit? (15 points)

11. If $z = x + iy$ is a complex number such that $x^2 + y^2 \neq 0$ find x and y such that $z^2 = i$. (15 points)