

Name

1. Assuming only the axioms for counting numbers and the definitions of addition and multiplication, state and use induction to prove the commutative and associative laws for counting numbers. (25 points)
2. State the definition of the relation $\frac{a}{b} \sim \frac{c}{d}$ that defines rational numbers. Prove that the right distributive law holds for rationals. (25 points)
3. Suppose z is a base n name for the number $(z)_n$.
 - (a) Give a rule for deciding when the number $(z)_n$ defined by z is bigger than the number $(w)_n$ defined by w .
 - (b) Prove that every whole (or signed) number has exactly one proper name.(25 points)
4.
 - (a) Use the Euclidean algorithm to find the greatest common divisor of 427 and 64.
 - (b) Using the previous part, find the continued fraction expansion of $427/64$
 - (c) Draw the Farey Diagram and locate $4/3$ and its nearest neighbors; what are they?.
 - (d) Use the diagram to find the continued fraction expansion for $4/3$ and the continued fraction expansion for each of its neighbors.
 - (e) Show how these continued fractions are related.(25 points)