Mat 237/637 Sample Optional Exam

May 13

Name

1. (10 pts) Prove that if x is irrational then $\sqrt{x}$ is irrational
2. (10 pts) Let $S = \{a, b, c, d, e, f\}$. How many different functions are there from S to itself that are both one-to-one and onto?
3. (15 pts) Let $T = \{2, 4, 7, 9, 27, 29\}$ and suppose you want to see if 17 is in the list.
   (a) Give the steps of the linear search algorithm to solve this problem
   (b) Give the steps of the binary search algorithm to solve this problem.
   (c) Can you tell whether one algorithm is better than the other?
4. (15 pts)
   (a) Define what it means for a and b to be congruent modulo 7.
   (b) Which pairs of the integers $-11, -8, -7, -1, 0, 3$ and 17 are congruent modulo 7?
   (c) Show that if a and b are congruent modulo 7, then $10a + 13$ and $-4b + 20$ are also congruent modulo 7.
5. (10 pts) Show that
$$1^3 + 3^3 + 5^3 + \ldots + (2n + 1)^3 = (n + 1)^2(2n^2 + 4n + 1)$$
   whenever n is a positive integer.
6. (10 pts) Use mathematical induction to show that $2^n > n^3$ whenever n greater than 9.
7. (10 pts) How many license plates can be made using either two letters followed by four digits or two digits followed by four letters.
8. (20 pts) How many ways are there to choose 10 items from 6 distinct boxes when
   (a) the items in the choices are ordered and repetition is not allowed?
   (b) the items in the choices are ordered and repetition is allowed?
   (c) the items in the choices are unordered and repetition is not allowed?
   (d) the items in the choices are unordered and repetition is allowed?