

**Name**

Part I: 36 points

1. What are the universal and existential quantifications of the predicate  $P(x)$ ? What are their negations?
2. What does it mean for two propositions to be logically equivalent?
3. Define the intersection of a pair of sets
4. Give an example of a function from the set of positive integers to itself that is neither one-to-one or onto
5. Describe the insertion sort algorithm
6. Define the least common multiple of two integers

Part II: 64 points

1. Find the truth table for the compound proposition  $(p \wedge q) \rightarrow (p \vee \neg r)$ .
2. State the contrapositive of the implication "If it rains today, then I will drive to work"
3. Express the following statement using quantifiers: "No student in this class has taken all courses in every department in the school of mathematical sciences."
4. Let  $A$  be the set of English words that contain the letter  $x$  and  $B$  the set of English words that contain the letter  $q$ . Express the following as a combination of  $A$  and  $B$ :
  - (a) The set of English words that contain neither an  $x$  and a  $q$ .
  - (b) The set of English words that contain the letter  $q$
5. Let  $f$  be a function from the set  $A$  to the set  $B$ . Let  $S \subset A$  and  $T \subset B$ . Show (by example) that if  $f$  is not one-to-one, the following is not true:  $f(S \cap T) = f(S) \cap f(T)$ .
6. Use the binary search algorithm to decide if 9 is in the set  $\{2, 3, 4, 7, 8, 11, 13, 15, 16, 21, 25, 30\}$  (Write out the steps in the search)
7. Explain why  $n \text{ div } 7$  equals the number of weeks in  $n$  days
8. Are the pair  $(253, 121)$  relatively prime?