

Name

1. State the five axioms for the counting numbers. (20 points)
2. Assume that for any counting number x , $1 + x = x^+$. Prove by induction that for all counting numbers $x + y = y + x$. (20 points)
3. Assuming the commutative, associative and distributive laws of addition and multiplication, prove that $(4 \cdot 5) \cdot (8 \cdot 9) = [(9 \cdot 5) \cdot 4] \cdot 8$ (10 points)
4. Use the definition of exponentiation to compute 4^3 . (10 points)
5. State the Well ordering principle. (15 points)
6. Consider the theorem “For any counting numbers x, y, z , $x > y$ if and only if $x \cdot z > y \cdot z$.” Using the trichotomy law and the “if” statement prove the “only if statement”. (15 points)
7. Using the definition of subtraction show that $8 - 5$ is defined. Using the definition of addition (using successors) show that $8 - 5 = 3$. (10 points)