Homework Feb 13, 2007

1. Each of the following functions undergoes a bifurcation of fixed points at the given parameter value. Use either algebraic or graphical methods to identify this bifurcation as either saddle node or period doubling or neither. In each case sketch the phase portrait for typical parameter values below, at and above the bifurcation value.

   (a) $F_\lambda(x) = x + x^2 + \lambda, \lambda = -1$

   (b) $G_\mu(x) = \mu + x^3, \mu = -1$

   (c) $S_\mu(x) = \mu \sin x, \mu = 1$ item $H_c(x) = x + cx^2, c = 0$

   In the next two problems let $Q_c = x^2 + c$.

2. Prove that the cycle of period 2 for Q_c given by the points $q_\pm = \frac{1}{2}\big(-1 \pm \sqrt{-4c-3}\big)$ is attracting for $-\frac{5}{4} < c < -\frac{3}{4}$.

3. Prove that this cycle is repelling for $c < -\frac{5}{4}$.

   In the next three problems let $F_\lambda(x) = \lambda x(1-x)$.

4. For which values of λ does F_λ have an attracting fixed point at $x = 0$?

5. For which values of λ does F_λ have a nonzero attracting fixed point?

6. Describe the bifurcation when $\lambda = 3$.

7. Consider the family of functions $F_\lambda(x) = x^5 - \lambda x^3$. Discuss the bifurcation of 2-cycles that occurs when $\lambda = 2$. *Hint*: Note that this is an odd function for each λ, so points of period 2 may be found by solving $F(x) = -x$.