

Sample essay questions for the final exam. Answers should be approximately 3 handwritten pages. They should be written in proper English and accompanied by relevant graphs or pictures.

1. In an essay discuss the notion of conjugacy. Why is it important in dynamical systems? In the essay you should show explicitly that the functions  $f(z) = z^2$  on the unit circle and  $g(z) = z^2 - 2$  on the interval  $[-2, 2]$  are (semi-) conjugate.
2. Discuss the concept of a filled Julia set for the quadratic function  $Q_c(z) = z^2 + c$ . Discuss the escape criterion for these functions. Include a clear discussion of the Fundamental Dichotomy. Give a brief description of the proof that, if the orbit of 0 escapes, then the filled Julia set consists of infinitely many pieces.
3. Discuss the role of the critical orbit (the orbit of 0) in the dynamics of  $Q_c(x) = x^2 + c$ . Include both the real and complex case.