

Exercises on Continued Fractions and Geometry

1. Check whether the following are neighboring pairs of Farey numbers:

$$(4/3, 3/2) \quad (7/12, 8/11) \quad (23/5, 39/7)$$

2. Draw the Farey diagrams to check your answers to the above
3. Use the Euclidean algorithm to find all neighbors of $43/5$
4. Draw the fan to illustrate the answer the the above
5. Find a neighbor p/q of $4/7$ that is closer than $1/1000$ to it. That is find a neighbor p/q end such that $|p/q - 4/7| < 1/1000$
6. Find the rightmost and leftmost neighbors of:

$$3/8, 4/5, 7/12, 7/4$$

and draw the corresponding umbrellas

7. Find the continued fractions for

$$5/8, 13/37, 2/23, 9/13$$

8. Draw the diagrams for finding the continued fractions of

$$5/4, 4/3, 9/7$$

Can you see their relation as Farey numbers reflected in the diagram and in the continued fractions?

9. Find the continued fraction expansion for $x = 1 + \sqrt{3}$ using the method of the notes.
10. Show that for $x = 1 + \sqrt{3} = [n_1, n_2, \dots]$, we have $[n_1, n_2, \dots, n_{2k-1}] < x < [n_1, n_2, \dots, n_{2k}]$
11. Show that $x[n_1, n_2, \dots, n_{2k-1}]$ is a least upper bound for the continued fractions $[n_1, n_2, \dots, n_{2k-1}]$ (the ones with an odd number of terms) and a greatest lower bound for the even ones $[n_1, n_2, \dots, n_{2k}]$.